A mathematical exploration into manipulation and control of a bifurcative dreaming process

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Summary. Mathematical models derived from the concepts of nonlinear dynamics and chaos theory for representing the highly nonrational transformations and bifurcations in a dreaming process may be highly useful for the study of dreams. In this work, three chaotic dynamical models known as the logistic map, sine map and cubic map have been adopted as a mathematical analogy for representing bifurcative dreaming processes. Further, the effect of a dream potentiator has been incorporated into the adopted models. For studying the effects of the dream potentiator on the actual dreaming process, computer simulations were performed. Results demonstrate that the dream potentiator may enhance the actual dream by elevating the dream states, by expanding the bifurcations and by reducing the number of successive steps required for attaining a dream state of deterministic chaos.

Keywords: Bifurcating dreams, chaos theory, mathematical models

"However, if we do discover a complete theory, it should in time be understandable in broad principle by everyone, not just a few scientists. Then we shall all, philosophers, scientists, and just ordinary people, be able to take part in the discussion of the question of why it is that we and the universe exist."

-Stephen Hawking

1. Introduction

Dreams are the direct, natural expression of the current condition of the dreamer's mental world. The nature of dreams is to present a spontaneous self-portrayal, in symbolic form, of the actual situation in the unconscious (Jung, 1967). The nature of the dream-state is highly subjective and a truly personal experience making the scientific analysis of dreaming highly difficult. Dreams often contain material that is nonsensical and challenging to interpret rationally, making the characterization of dreams from an objective point of view a perplexing task (Franklin & Zyphur, 2005). Also, there is a significant difference between dreams experienced during sleep and dreams recollected after waking up. In a mathematical point of view, dreaming as subjective experiencing during sleep may be considered as a 'continuous system' and has a value at each and every point of time. However, dream reports which are a more or less adequate recollection of what happened during sleep after waking up, may be considered as a 'discrete or a sampled system' with values only at certain time points.

The nonrational transformations, divisions or bifurcations are typical examples of nonlinear dynamics of self organization in modern chaos theory (Rossi, 1989). The dreaming process is an active process of getting closer to a goal by a series of steps or successive approximations. In mathematical representation, the series of steps are known as iterations and recursions (Rossi, 1998). Ernest Lawrence Rossi, in his work, explains that the spontaneous bifurcating transformations in dream imagery are critical phase transitions iterating important physiological choice points and developments that provide conditions for self-reflection and the expansion of awareness and individuation (Rossi, 1998). An example of such a bifurcative dream is shown in Figure 1.

The series of steps, as described in Figure 1, is similar to a neurobiological calculation on an unconscious level, which is called as iterations or recursions in the mathematical language of nonlinear dynamics. Rossi explains that the branching pathways can be represented as a series of solutions to an equation obtained by a process of feedback or iteration and each branch represents a 'choice' or an answer. All complex biological and psychological systems involve multiple pathways and processes of feedback that can be illustrated using nonlinear mathematical equations (Rossi 1998).

For a bifurcative dreaming process, as shown in Figure 1, it is possible to obtain a mathematical analogy using certain dynamical systems which exhibit similar chaotic and bifurcation behavior comparable to the original dreaming process, as proposed by (Rossi, 1997). Such chaotic systems may provide a meaningful mathematical model of the nonlinear pathways of consciousness in general and dreams in particular (Rossi, 1997; Rossi, 1998; Rossi, 1999). This mathematical analogy of dream bifurcations is obtained from the fact that some equations have multiple solutions. For example, the simple equation $x^2=9$, produces two solutions x=+3 and x=-3 (Rossi, 1998). Further, more complex and successive bifurcations can be represented using multiple solutions obtained from chaotic nonlinear equations such as the logistic map, sine map, cubic map etc by the process of feedback or recursion.

Figure 2 shows the analogy between the actual dreaming process and its mathematical representation, using the

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Figure 1. A typical bifurcative dream.

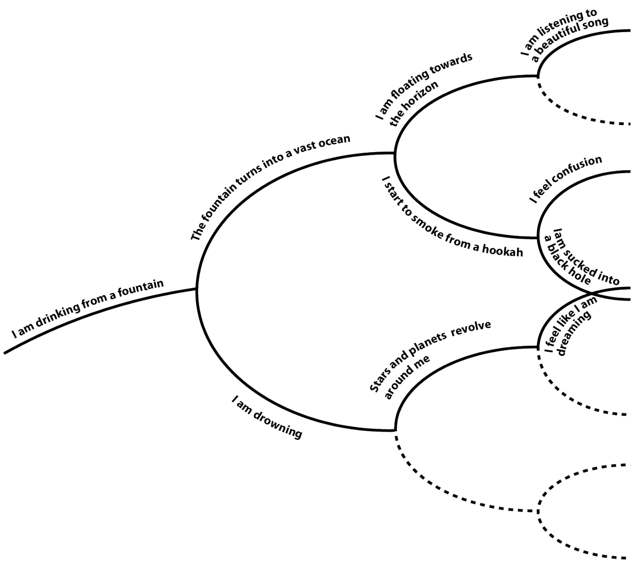
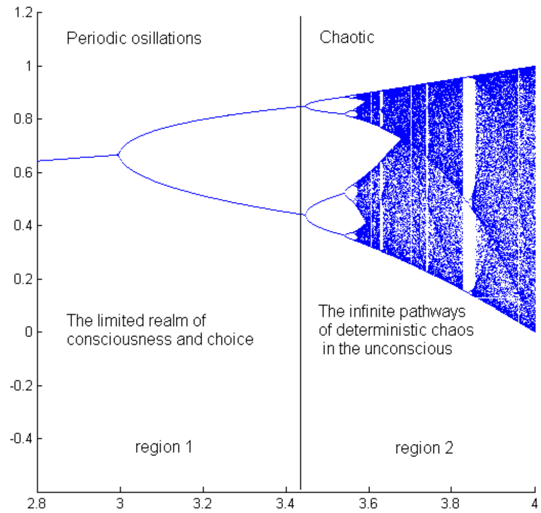


Figure 2. The analogy between the actual dreaming process and its mathematical representation.



bifurcation diagram obtained from a common chaotic system known as the logistic map (Rossi, 1998; Alligood, 1997). Mathematically, region 1 representing an oscillatory region can be used to describe the limited realm of consciousness and choice and region 2 can be used to describe the infinite pathways of deterministic chaos in the unconscious (Rossi, 1998). Also, it might be possible to obtain analogies for the concepts of dream within a dream using the overlapping bifurcations in Figure 2.

Dreams can be described in terms of specific neurochemical events or changes in neurotransmitter levels such as dopamine, which take place within the brain (Yuschak, 2006). There are several drugs which can affect the neurochemical events in the brain and such drugs can effectively induce a dream or may enhance a dreaming process (Pagel & Helfter, 2003; Toro & Thomas, 2007). There are several reports on specific drugs such as Galantamine, which induce or enhance the dreaming process (Yuschak, 2006). Such a substance is referred as a dream potentiator. In this article, the dreaming process induced using a dream potentiator, is described as a forced dreaming process and dreaming process without the influence of a dream potentiator, is referred to as an unforced or a spontaneous dreaming process.

The objective of this work is to develop mathematical models for analyzing the effects of dream potentiators on bifurcative dreaming processes.

2. Methodology

In this study, three different dynamical systems exhibiting chaotic and bifurcation behavior, namely the logistic map, sine map and cubic map (Alligood, 1997; Devaney, 1989; Strogatz, 1994) represented by equations (1), (2) and (3) respectively, have been used for obtaining a mathematical analogy of dream bifurcations. These models were chosen as a mathematical analogy of actual bifurcative dreaming processes because such chaotic models exhibit similar bifurcation behavior when compared to the actual dreaming process.

(1) $x_{n+1} = Ax_n(1-x_n)$

(2) $x_{n+1} = A\sin(\pi x_n)$

(3) $x_{n+1} = Ax_n(1-x_n^2)$

Consider that the above mentioned systems describe an unforced of autonomous dreaming process. In order to analyze the possibility of manipulating or controlling a bifurcative dreaming process, let us include the effect of a forcing function (un) in equations (1), (2) and (3), assuming that the forcing function is the effect of a dream potentiator.

Figure 3. Computer simulations of an autonomous dreaming process, and forced dreaming process (with an additive forcing function) obtained using the logistic map considered as a mathematical analogy of the bifurcative dreaming process.

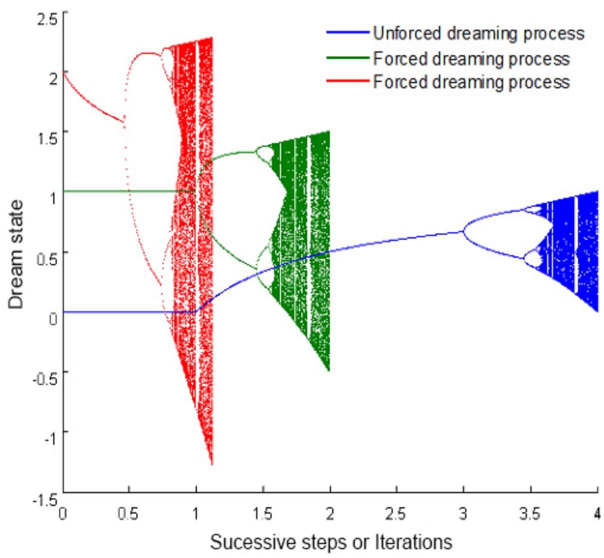
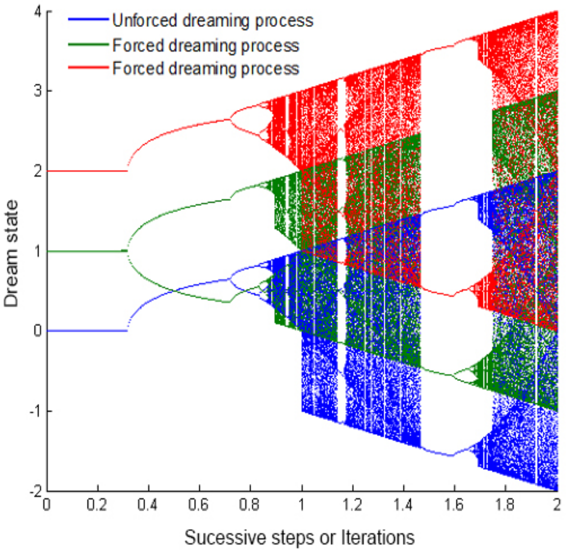


Figure 4. Computer simulations of an autonomous dreaming process, and forced dreaming process (with an additive forcing function) obtained using the sine map considered as a mathematical analogy of the bifurcative dreaming process.



Since, we do not know how the dream potentiator affects the actual dreaming process, let us consider two different scenarios:

i. The dream potentiator affects the actual dreaming process in an additive manner, as shown in equations (4), (5) and (6).

(4) $x_{n+1} = Ax_n(1-x_n) + u_n$

(5) $x_{n+1} = A\sin(\pi x_n) + u_n$

(6) $x_{n+1} = Ax_n(1-x_n^2) + u_n$

ii. The dream potentiator affects the actual dreaming process in a multiplicative fashion, as shown in equations (7), (8) and (9).

(7) $x_{n+1} = Ax_n(1-x_n) \times u_n$

(8) $x_{n+1} = A\sin(\pi x_n) \times u_n$

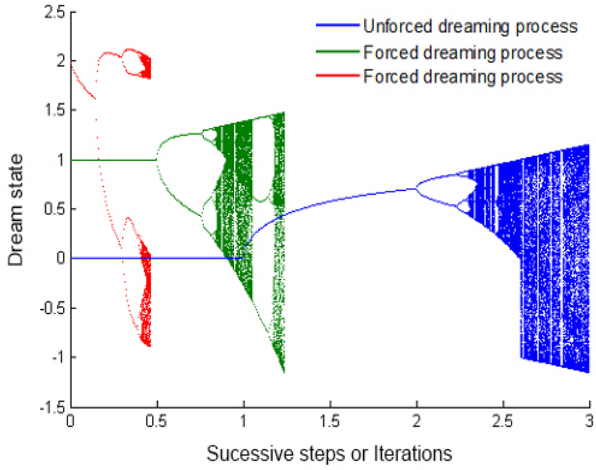
(9) $x_{n+1} = Ax_n(1-x_n^2) \times u_n$

The computer simulations for studying the effects of the dream potentiator were performed using Matlab 7.0.1 on a computer with 1.73 GHz processor.

3. Results and Discussion

The computer simulations of an autonomous or unforced dreaming process, and forced dreaming process with an additive forcing function, obtained using the logistic map considered as a mathematical analogy of the bifurcative dreaming process, is shown in Figure 3. It is seen that the effect of the dream potentiator expands the dream state (described by the green lines in the Figure). Further, increasing the concentration of the dream potentiator results in a

Figure 5. Computer simulations of an autonomous dreaming process, and forced dreaming process (with an additive forcing function) obtained using the cubic map considered as a mathematical analogy of the bifurcative dreaming process.



reduced number of iterations or successive steps, required for achieving a chaotic dream state (described by the red lines in the Figure).

Figure 4 shows the computer simulations of an autonomous or unforced dreaming process, and forced dreaming process with an additive forcing function, obtained using the sine map considered as a mathematical analogy of the bifurcative dreaming process. It can be seen that the effect of the forcing function or the dream potentiator elevates the state of the dream.

Similarly, Figure 5 shows the computer simulations of an autonomous dreaming process, and forced dreaming process with an additive forcing function obtained using the cubic map considered as a mathematical analogy of the actual bifurcative dreaming process. In this case, it is seen that by

Figure 6. Computer simulations of an autonomous dreaming process, and forced dreaming process (with a multiplicative forcing function) obtained using the logistic map considered as a mathematical analogy of the bifurcative dreaming process.

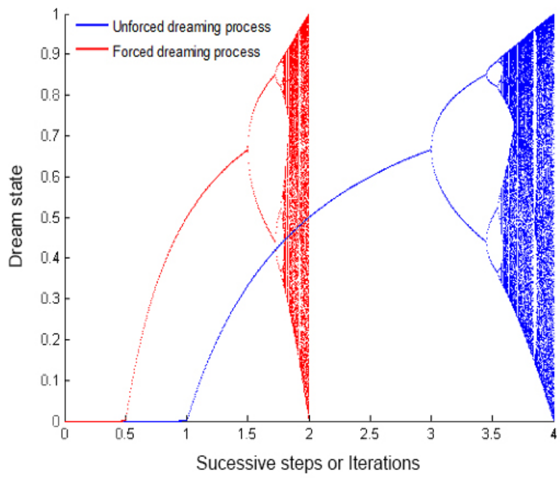


Figure 7. Computer simulations of an autonomous dreaming process, and forced dreaming process (with a multiplicative forcing function) obtained using the sine map considered as a mathematical analogy of the bifurcative dreaming process.

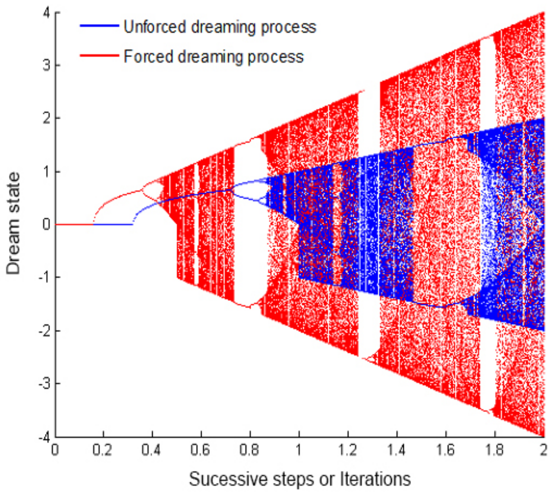
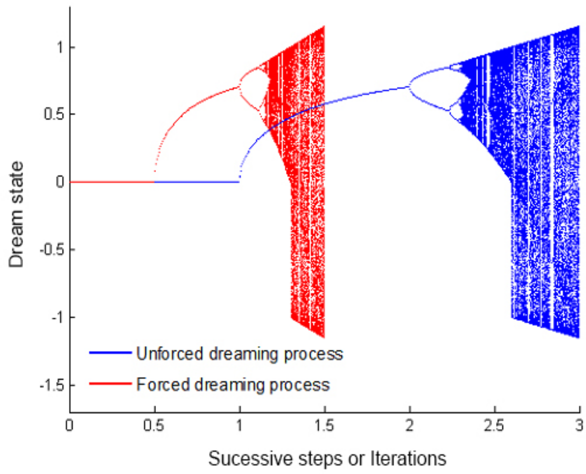


Figure 8. Computer simulations of an autonomous dreaming process, and forced dreaming process (with a multiplicative forcing function) obtained using the cubic map considered as a mathematical analogy of the bifurcative dreaming process.



introducing a dream potentiator, it is possible to achieve the chaotic dream state within fewer iterations. Further increase in the effect of the dream potentiator results in a reduced number of iterations, required for attaining a chaotic dream state.

Figure 6 shows the computer simulations of an autonomous dreaming process, and forced dreaming process with a multiplicative forcing function obtained using the logistic map considered as a mathematical analogy of the actual bifurcative dreaming process. It appears that, the dream state of deterministic chaos is achieved with less number of successive steps without expansion of the dream state, if the effect of the dream potentiator is multiplicative in nature.

The computer simulations of an autonomous dreaming process, and forced dreaming process with a multiplicative forcing function obtained using the sine map considered as a mathematical analogy of the bifurcative dreaming process, is presented in Figure 7. Results demonstrate that the dream potentiator may enhance the dreaming process with a large number of bifurcations.

Finally, Figure 8 presents the computer simulations of an autonomous dreaming process, and forced dreaming process with a multiplicatve forcing function, obtained using the cubic map as a mathematical analogy of the bifurcative dreaming process. In this case, it is seen that the multiplicative effect of the dream potentiator results in fewer recursions or iterations required for attaining the pathways of deterministic chaos in the dream.

4. Conclusion

To conclude, it appears that it may be possible to manipulate or control a bifurcative dreaming process using a suitable dream potentiator. The effects of the dream potentiator may enhance the dreaming process by increasing the number of bifurcations, or elevate the dream state, or lessen the number of successive steps required to attain a dream state of deterministic chaos.

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