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On the Mathematical Connections of Ancient Measures of Length

There is not doubt that the measures of length once were derived from the human body: the cubit, the foot, the palm and the digit. From that follows that, at the very beginning of measuring there was no simple way to say how many feet ten cubits are or how many palms one foot and so on. But it is sure, that at **one** point in history there was a definitively given system of permutation of one unit into the other; for instance that one foot is made up of four palms and again one palm is made up of four digits.

I want to deal with the time between those two historical points. It can be taken for proved, that at the time of the Great Pyramid (2600 B.C.) there was a well established system of units of length[1]. But it is quite uncertain, how early man begun to measure.

With the developpement of clay bricks however the builders of houses and palaces were forced to work more exactly because of the large number of bricks needed and for the sake of easy and durable repair of a building. In Tepe Yaḥyā we can trace regularily made bricks and rooms very early[2] and we can obtain a preliminary mean value for a cubit.

Tepe Yaḥyā 5.–4. millennium B.C.

bricks (6 x 12 in)	cubit	506.7 mm
		508.0
bricks (9.5 x 9.5 x 4.75 in)		517.1
clay tablets (2 x 1.33 in)		508.0
bricks (17; 14 in)		518.16
		520.39
room (9 x 24 ft)		514.35
		522.5
room (5 x 5 ft)		508.0

mean 513.7 mm

Fig. 1

In Uruk/Warka the situation is a little bit better due to the good preservation of stone monuments which could not be erected without a system of measurement. They are more easily to measure today[3].

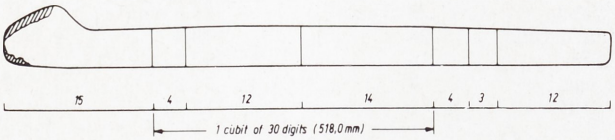
URUK 4. millennium B.C.

'Lime'- temple	75 x 29 m	cubit	520.83 mm
			517.85
'Pillar'- temple	ϕ 2.62 m		517.1
Temple C bricks	16 x 6 cm		516.6
buildg.	55 x 22 m		518.87
			517.65
Temple D	83 x 53 m		518.75
			517.58

mean 518,15

Fig. 2

But the best proof is a measure rod of copper found in a temple of Nippur (published by *Unger*[4]). It dates from the 1st half of the third millennium or even earlier. All over it is 4 feet long (1106,36 mm!) and has some subdivisions into the most usual units. As a mean value of those 518.6 mm for the cubit is found.



1 Elle zu 30 Zoll (518,0 mm)

Fig. 3

The uncertainity appears in the fourth position.

NIPPUR late 4. or early 3. mill. B.C.

Measure rod of copper with cubit & other subdivisions

cubit	=	30 dig.	518,6 mm
foot		16	276.59
palm		4	69.15
digit		1	17.29

Fig. 4

From the early dynastic times in Egypt we can obtain a few measurements from stone buildings[5]. The mean value happens to be exactly the unit of Nippur, but this accuracy is not obtainable.

1 Paulsen 1969.
2 Lamberg-Karlowsky 1971.
3 Mallowan 1962.
4 Unger (1916) 1927.
5 Iskander–Badawy 1965.

EGYPT early 3. millennium B.C.

Pyramid of Djoser, wall (550 x 280 m)	cubit	518.87 mm
		518.51
–, window (30 x 13 cm)		524.7
Pyr. of Sekhenkhet (536 x 194)		515.4
		518.7
'foot of Djoser' after MYERS (274.9 mm)		515.44

mean: 518.6 mm

Fig. 5

But *Flinders Petrie* reports a measure rod of Djoser, which shows a unit of 520.54 mm, slightly deviating from the cubit of Nippur[6].
While at this time (III. Dynasty) the cubit is subdivided into 30 parts, later the cubit was divided in 28 parts (= digits). This is known as well from the Great Pyramid as from the frequent measure rods of the New Kingdom[7].

EGYPT about 2000 B.C. and younger

Several measure rods, mean value

cubit	=	28 digits	523.6 mm
small cubit		24	448.8
foot		16	299.2
palm		4	74.8
digit		1	18.7

Fig. 6

But also the length of the cubit is enlarged to 523.6 mm, first found at the Great Pyramid. There are more than six different cubit rods from the New Kingdom, from which the length of the cubit can be taken very exactly. The mean value is 523.6 mm[8].
Flinders Petrie learnt from the old Egyptian texts, that there was used at the same time another unit of length, called 'remen'. It was not independent from the Royal Cubit mentioned before. It was used by the surveyor in field measurement. The connection of Royal Cubit and remen is given by the Pythagorean proposition. Let one Royal Cubit be the side of a square. Then the diagonal is equal to two remen. The remen is reported to have 5 palms or 20 digits.

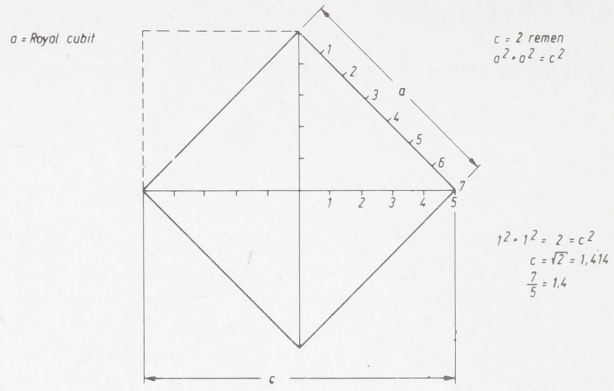


Fig. 7

One may put the question, how long the cubit may be, which is made up of 7 palms or 28 digits which come from the remen. (Apparently this digit is shorter than that of the Royal cubit.) The answer is surprising: The cubit derived from the remen is 518.3 mm long and thus equal to the cubit from Nippur. If one takes into account that the cubit rod of Djoser was 520.54 mm, then the cubit derived from the remen seems to be older than the Royal Cubit. From that follows the Royal Cubit to be the diagonal of a square with the side length of one remen.
Even more surprising is the length of one digit of the remen measure. It is 18.5 mm long and absolutely equal to the digit of the Roman system of measurement.

EGYPT about 2500 B.C.

Calculated from the 'remen'

cubit	=	28 digits	518.6 mm
foot		16	296.3
remen		5	92.6
palm		4	74.1
digit		1	18.5

Fig. 8

The most surprising fact in this connection is indeed, that 16 cubits of Nippur are equal to 10 of those Megalithic Yards, which are found by *Thom – Thom* to be the base of the measuring system used by the builders of the megalithic monuments in Great Britain and the Britanny[9].

[6] Flinders Petrie 1934.
[7] Skinner 1957.
[8] Myers 1966.
[9] Thom – Thom 1972.

THE MEGALITHIC YARD about 2500 B.C.

10 MY	= 4 x 4 = 16 cubits of Nippur
	4 x 4 x 518.3 mm = 8292.8 mm
8293.0 mm ≈	4 x 4 x 518.6 mm = 8297.6 mm

Fig. 9

It is laid stress on the fact that the figures used in the calculations are dericed from real rods in the case of Nippur, Egypt and the Roman Empire. The calculation of the remen is given by ancient Egyptian texts. Solely the length of the Megalithic Yard is the result of a statistical evaluation. It may be added that *R. Müller* found a almost equal figure for formerly German megalythic buildings.

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