MOLDMATE IDENTIFICATION IN PRE-19TH-CENTURY EUROPEAN PAPER USING QUANTITATIVE ANALYSIS OF WATERMARKS, CHAIN LINE INTERVALS, AND LAID LINE DENSITY

SARA F. GORSKE, C. RICHARD JOHNSON, JR., WILLIAM A. SETHARES, MARGARET HOLBEN ELLIS, PAUL MESSIER

ABSTRACT | Handmade laid paper has the important quality that every sheet of paper formed on the same papermaking mold retains a nearly identical imprint of the mold's wire structure. These "moldmates" are identified by analyzing the recorded wire features, which are visible using transmitted light. When visual analysis is not sufficient to distinguish moldmates, three features of the mold's wire mesh can be quantitatively analyzed using image processing techniques: watermark shape and placement, chain line intervals, and laid line density, for which a new method of analysis is introduced here. Using signal processing procedures, the frequency of the laid lines across a sheet of paper was found to fluctuate in a pattern unique to that mold. These quantitative methods were tested on a sample set of blank sheets of laid paper which had been inserted into a 1536 edition of *De re militari* by Vegetius; computational analysis using any one of the three features was able to distinguish between four molds used in the group of papers. These results demonstrate that any of these techniques can be chosen as appropriate to determine moldmates from within a set of laid paper, regardless of size or the inclusion of a full watermark.

KEYWORDS | signal processing, quantitative methods, feature extraction, mnemosyne, historic paper

Introduction

Paper is one of the most transformative materials to have impacted human existence. Paper has served as the medium on which history has been recorded in its mundane and forgotten minutiae as well as its vivid and extraordinary details. While the written, drawn, or printed marks left on the page can narrate a story of history and craft, the specific physical characteristics of every sheet of paper are just as descriptive. A paper's thickness, appearance, and material composition are essential aspects for reconstructing the tale of paper production and use throughout history.

Before the 19th century and the introduction of papermaking machines, European paper was made by hand. To manufacture what is often described as "laid" paper, a mold, or a wooden frame supporting a mesh of wires arranged in perpendicular directions, was dipped into a vat of suspended pulp. Water would drain through the wires while the macerated fibers remained atop the mesh[1]. The resultant sheet of paper retains an imprint of the mold structure, recording the locations of the wires and their placement relative to each other. Several of these wire features are of interest: the laid lines, which give laid paper its name, are densely packed and parallel to the long edge of the wooden frame, while the chain lines are more widely spaced and were created by lacing thinner wires perpendicularly through the laid wires to keep them in place on the mold. Additionally, most papermakers added a watermark to the mold. Watermarks were created by twisting a thin wire into a simple, often pictorial, shape and securing it to one side of the frame of laid and chain wires. The watermark is often joined by a simpler counter mark on the opposite side. Because the paper is thinner in the vicinity of wire features, which impeded the flow of fibers, the locations and shapes of those features are preserved and visible when light passes through the sheet.

Because each papermaking mold was made by hand, no two molds are exactly alike. Papers made on different molds therefore will not have identical watermarks or patterns of laid and chain lines, although these differences can be slight. In contrast, every paper created on the same mold retains a nearly identical imprint of wires to its moldmates.

Figure 1. Left (with 'PR' countermark) and right (with foolscap watermark) sides of a folded sheet from De re militari *by Vegetius, published 1536. The mold was characterized as mold D based on visual inspection of the countermark and watermark*.

By identifying and classifying the subtle differences on laid papers, their similarity and potential identical nature may be determined, allowing historians to gain insight into the date and location of a sheet's origin. Such information is useful in the construction of chronologies both for individual artisanal and artistic practices and for broad industrial patterns of distribution and use.

These questions have been applied to some of the most famous names in art and literature, from authenticating and reassembling Shakespeare First Folios[2] to dating individual prints pulled from Rembrandt's more than 300 copperplates[3]. Much of the current research relies upon identifying papers by means of watermarks, usually through a qualitative comparison[4], though computer-based techniques have been tested[5]. Unfortunately, watermarks were generally placed in one location on a mold, which could measure up to four-and-a-half square feet in area[6]. In practice, the full mold-sized paper was often cropped to more desirable dimensions, often eliminating the watermark entirely or leaving behind only a fragment. As a result, only a fraction of historic works retain watermarks (an estimate for Rembrandt prints is only one-third), limiting the number of matches that can be found within watermark image databases.

Recently, computer-aided methods based on images have been used to extract and record data about paper structure and formation[7]. The promise of this technique is twofold. First, the image-based methodology allows many more works to be examined in relation to each other than would be possible were the artifacts to remain in scattered private and public collections. Second, the character of laid paper is such that the positions of small but unique features can be extracted and analyzed using signal processing techniques when the human eye might not be able to detect differences with confidence. While the implementation of these methods is limited by the quality of data (heavily-inked surfaces are often impenetrable to light and can interfere with the detection of the underlying density variations, and the small scale of the wires requires high resolution imaging[8]), wire frequency has long been recognized as a potential finger-printing method for paper molds. Software-based methods have the potential to obtain precise, quantitative data on the position and frequency of wires, allowing data to be compared between papers to discover potential moldmates.

Recent scholarship has introduced several techniques for measuring the chain line intervals around watermarks[9], but for small papers or low resolution images, few chain lines are visible and measurable, leading to a reduction in data set size and a loss of uniqueness in the spacing patterns. As a result, moldmates cannot always be identified with certainty. Extending wire frequency detection to the laid lines demonstrates much promise in discovering and classi-fying moldmates. Traditionally, since many images have not had sufficient resolution to determine laid line position

Figure 2. Watermarks appearing in the blank pages bound into De re militari*, from left to right: molds A, B, C, and D.*

precisely, the lines have only been measured to estimate a mean frequency, which can narrow the number of potential matches between sheets but cannot specifically determine differences between molds[10].

A full quantification of the laid line density would be able to characterize the variation across the entire surface of a sheet. Not only could the laid lines be sampled at many more locations on a paper surface, but their much greater quantity would also provide more points of comparison, such that a few erroneous measurements are much less capable of skewing an entire data set. Because the laid wires were arranged in a mold before the chain wires were secured, the laid lines can also vary in angles running roughly perpendicular from the chain lines. Further, the density of laid lines changes both parallel and perpendicular to the chain lines, such that each mold should have a unique two-dimensional laid line density profile, to which smaller cropped sheets from the same mold can also be matched.

Sample Preparation

A group of blank sheets that had been inserted into a 1536 French edition of *De re militari* by Vegetius were provided for study by the Conservation Center, Institute of Fine Arts, New York University. The blank sheets, while not contemporaneous with the pages printed upon for publication, were produced using 16th-century techniques for handmade laid paper and thus have all the relevant inner structural features (watermarks, chain lines, and laid lines). These pages comprised sixty-nine unbound and folded full-mold sized sheets and nine detached half-sheets; an example is shown in Figure 1. The sheets were in excellent condition, with very little discoloration or surface wear. As a result, their watermarks were visible, and the sheets were identified as belonging to four distinct moldmate groups through visual inspection of the five-pointed foolscap watermark and the countermark 'PR' appearing on each page. Six sheets of each proposed group were then imaged using transmitted light with a Nikon D810 camera with focal length 35 mm, aperture f/11, and shutter speed 1/3 s. The final images had 7360x4912 pixels, corresponding to 540 pixels per physical inch, or 212.5 pixels per physical centimeter, with the locations of the chain lines and laid lines visible.

Watermark Analysis

Visual analysis of the watermarks resulted in a tentative classification of the papers into four groups. Several features could be used to identify the marks, such as the shapes of the three roundels (the pyramid of circular shapes extending below the foolscap) at the base of the watermark or their distance from the adjacent chainlines (see Figure 2 for the preliminary division of watermarks into molds A, B, C, and D). Using these features allows for the creation of a theoretical decision table, pioneered in the analysis of Rembrandt watermarks by Johnson[11], for which a watermark should have a unique combination of yes/no answers for a series of qualitative questions about feature shapes and sizes. These questions, however, can fail to identify a watermark in two instances: cropped sheets with partial or fragmentary watermarks, and extremely similar watermarks which cannot be distinguished using the provided questions.

The watermarks of molds B and D illustrate the latter problem. While molds A and C can be distinguished visually due to the unique and noncircular curvature of the top roundel, the watermarks of molds B and D appear nearly identical and are more challenging to differentiate. Upon close viewing, the roundels of the mold B watermark appear be more centered within the chain lines than for mold D, but this result relies on side-by-side comparison of the two molds and thus is problematic for inspecting and identifying a single sheet. Features such as these can be quantified, however, by computing the distances between points on the watermark and the adjacent chain lines (Figure 3). The relative lengths between the marked points can be compared by using a simple ratio such as X_L/X_R (Figure 3 and Table 1) to measure how much smaller or larger one space is than another. Ratios are useful not only for the clear relationship between the visible features and the numbers but also because the unit of pixels is cancelled, eliminating any apparent differences in lengths due image variance in scale or resolution.

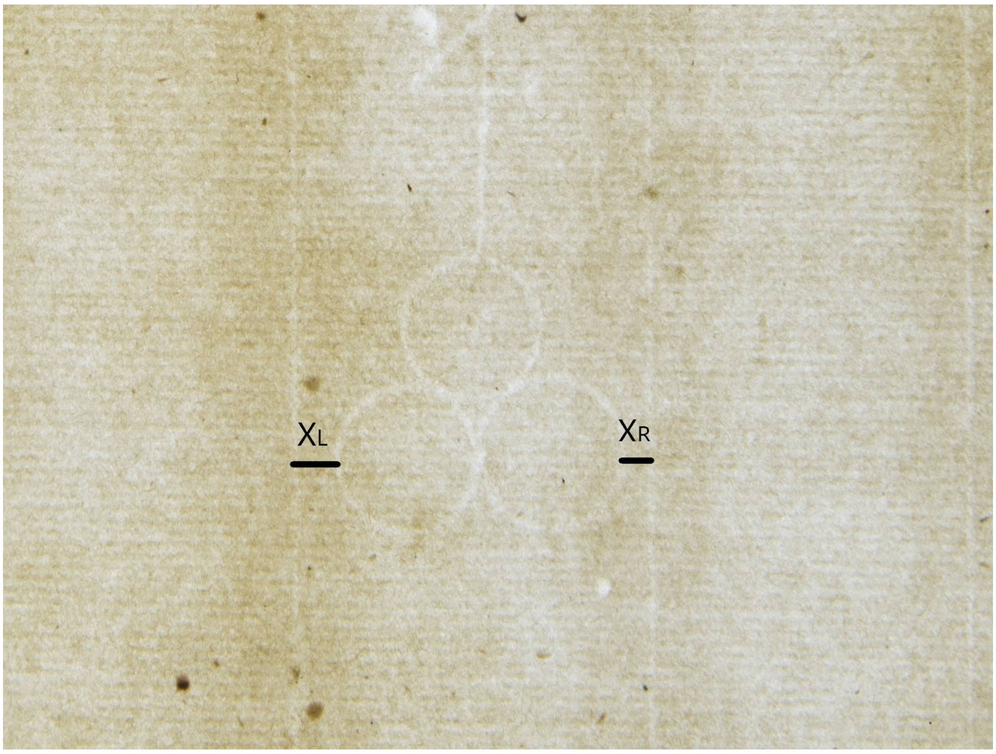

Figure 3. The roundels of the watermark of page 9 (mold D), with the spaces between the watermark's roundels and the nearest chain lines marked x_L and x_R as used in Equation 1.

MOLD B PAGE NUMBER	RATIO x_L/x_R	% DIFFERENCE
16	0.83	9.1
20	0.81	10.8
24	0.79	11.8
32	0.92	4.1
39	0.79	11.8
41	0.76	13.4

MOLD D PAGE NUMBER	RATIO x_L/x_R	% DIFFERENCE
7	1.61	23.5
9	1.53	21.1
18	1.50	20.0
29	1.67	25.2
37	1.65	24.6
45	1.60	23.1

Table 1. Comparison of the relative spacing between the roundels and the adjacent chain lines for molds B and D

Asymmetrical features, such as the face of the foolscap watermark, provide ready points of comparison between papers to identify individual molds. However, for watermarks that are essentially symmetric, or for fragmentary marks without an indication of orientation, such as the roundels of the foolscap only, a more robust method than the calculation of ratios is needed. The same locations on the chain lines and the watermark can be used, but the numerical comparison can be performed using the percent difference (Equation 1), using either space as x_L or x_R. The percent difference represents the amount by which the two intervals differ from each other, normalized by their total combined value, thus providing a result independent of orientation.

$$\% \text{ difference} = \frac{|x_L - x_R|}{x_L + x_R} \times 100\%$$

Equation 1

Using this method on the *De re militari* pages, a useful feature for distinguishing molds B and D is the distance between the widest part of the roundels and the adjacent chain lines to either side (Figure 3). This distance was measured on either side of the watermark, and both ratios and percent difference for six sheets of each mold were computed and are shown in Table 1. These values fall within non-overlapping ranges and readily differentiate between visually indistinct watermarks. Usefully, this form of analysis can be applied to any feature of the user's choosing, allowing the comparison of fragmentary watermarks to other fragments or full marks.

Chain Line Interval Calculations

A previously designed semi-automatic chain line marking software[12] was used to locate the chain lines on the paper and determine the intervals between each neighboring set of lines. The software converted these values a vector of values

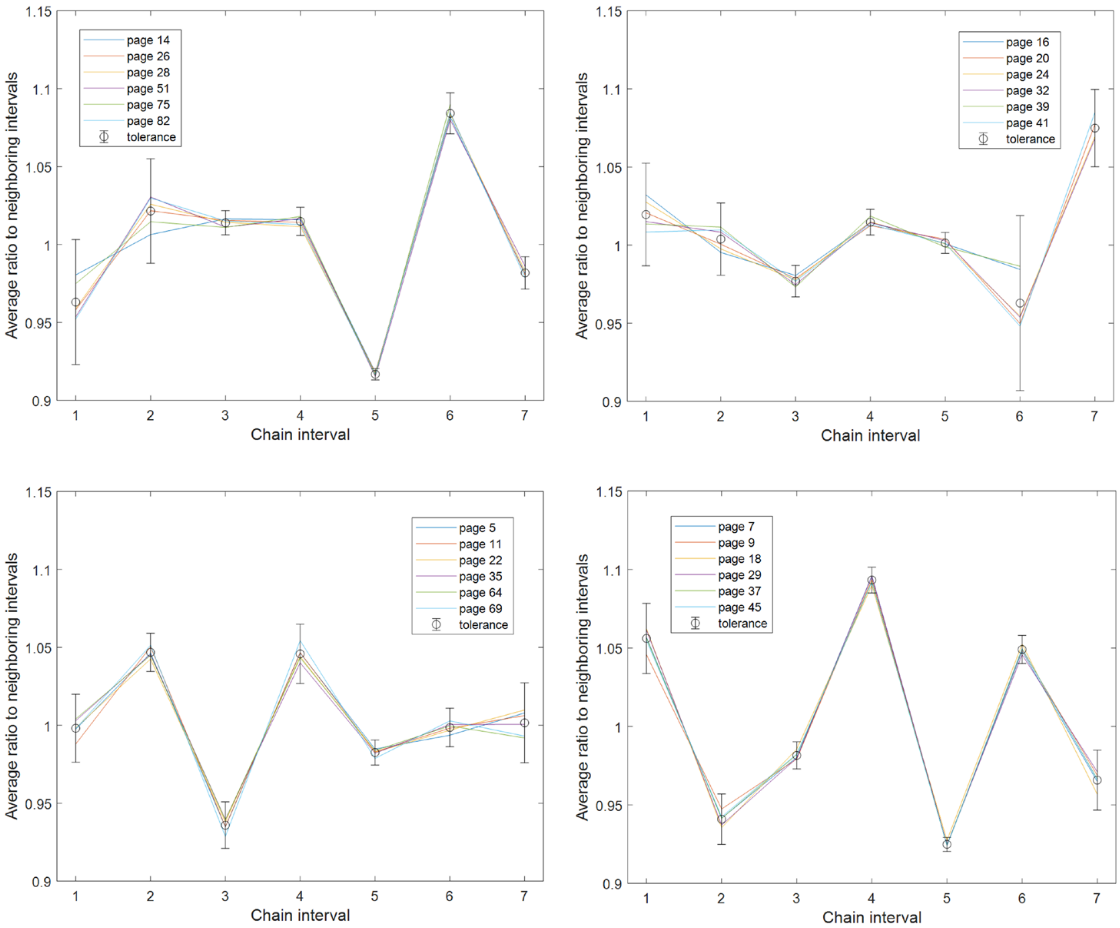

Figure 4. Chain line intervals and tolerances for the sides of the four molds which span watermarks. From left to right, top to bottom: molds A, B, C, and D.

and the width of each chain line space was normalized by its neighboring intervals. Each folio was confirmed to have a unique pattern of intervals, supported visually by the plots in Figure 4, which authenticated the identification of four molds within the set of sheets. Tolerance values were calculated by adding the standard deviation of the sample set for each interval to the maximum difference in ratios, allowing future samples to be matched to the correct mold. Comparing ratios rather than absolute values also negates the influence of any differences in the line spacings as a result of contraction over time or different image resolutions. Further, the representation of the varying chain line intervals using plots such as Figure 4 adds a new layer of visual analysis to the moldmate identification quandary, as compared to previous methods which relied solely on comparing numerical values.

Laid Line Detection and Mapping

The methods outlined in the previous sections pave the way for the development of software that automates moldmate identification. Such a program should be able to produce data for any imaged sheet of any size with minimal user interaction, the reliance on which is a limitation in the chain line marking software used in the previous section. Ideally, all visible features of a mold would be utilized, including chain line intervals as well as the density of laid lines, for which detailed data across the full area of a paper has not yet been achieved. Some work has been performed on this dilemma: previous work by van Staalduinen proposes using frequency domain methods to automate the detection of the laid line density[13]. However, the outlined method is limited in its application only to small areas of relevant papers and its use only of the average density of two sheets being compared as potential moldmates. Expanding van Staalduinen's approach by utilizing methods analogous to the canvas thread counting of Johnson, et.al.[14] and Sethares[15] can be used to produce detailed, quantified density data across the full area of a paper. The resulting technique can identify moldmates via application to full-sized sheets as well as cropped papers of arbitrary size.

A full mathematical description of the laid line detection process can be found in Appendix I and Figure 7. The resulting process outputs maps that describe the variance of laid line density across the surface of the paper using color and intensity variations. Figure 5 shows examples of the laid line maps for the sheets in Figure 2; a blue color indicates a smaller value while a red point has a higher density. For greater clarity, the maps show two halves of the mold, one containing the foolscap watermark and the other the opposing countermark. Within the set of tested sheets from *De re militari*, the mean laid line densities varied from 10.59 to 11.18 lines/cm. Table 2 shows these results for the sampled sheets from molds A and C, with full results across all molds shown in Appendix II.

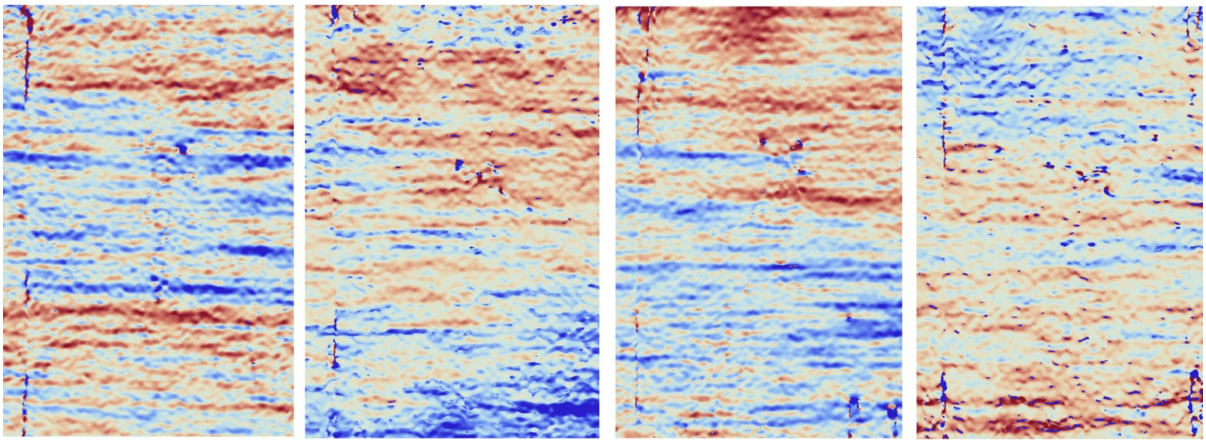

Figure 5. Laid line density plotted as intensity maps, with low densities in blue and high densities in red. From left to right, examples for the watermark sides of molds A, B, C, and D.

PAGE NUMBERS (MOLD A)	MEAN DENSITY (LINES/CM)	PAGE NUMBERS (MOLD C)	MEAN DENSITY (LINES/CM)
13-14	10.643	5-6	10.677
25-26	10.660	11-12	10.611
27-28	10.615	21-22	10.632
51-52	10.625	34-35	10.654
75-76	10.622	63-64	10.647
81-82	10.593	69-70	10.636

Table 2. Mean laid line density for selected pages of De re militari by Vegetius.

Discussion

For the *De re militari* pages, each mold is demonstrably unique using quantitative image and signal processing methods applied to any of the wire features studied (watermarks, chain lines and laid lines). In the case that full broadsheets or folios are present, as presented here, numerically characterizing any one of the wire features is sufficient to distinguish between papers made from the different molds and to identify moldmates.

Watermark analysis is potentially the most widely accessible method for full sheets of laid paper. In visual inspection, watermarks are the most readily identifiable features on paper and are often the only characterizable feature on images made with commonly employed techniques such as beta-radiography or low resolution digital photography, in which resolution or X-ray quality may be too low to locate laid or chain lines. If visual analysis is conclusive, then papers can be classified quickly, without the need to perform any manual or automatic quantification schemes. Within the *De re militari* sheets, for example, molds A and C can be distinguished without numerical analysis. In contrast, additional features are necessary to describe the difference between molds B and D. The method of measuring intervals between watermarks and chain line positions and calculating the resultant ratios achieves this distinction (Table 1); for the example feature of the spaces between the roundels and the adjacent chain lines, the simple ratios x_L/x_R fall between 0.76 and 0.92 for mold B and between 1.50 and 1.67 for mold D, while the percent difference is around 10% for mold B and 20% for mold D.

Ratio-based methods like percent differences have further advantages for the study of watermarks because of the opportunity to study fragments. In past studies, if the features that have been picked to identify watermarks are missing from a fragment, the sheet of paper containing the fragment may be neglected in further moldmate studies, even if other features are still visible. By deriving invariant ratios, however, cropped watermarks can be compared to each other as well as to full marks, expanding the watermark dataset available for study and moldmate classification.

Chain lines are also useful for classifying large sheets of laid paper and are essential for sheets that lack watermarks or countermarks. While the *De re militari* papers contain both a watermark and a countermark, some early molds lack countermarks, and additionally sheets were routinely trimmed, often eliminating part or all of a mark such that it cannot be studied with either traditional or quantitative watermark analysis.

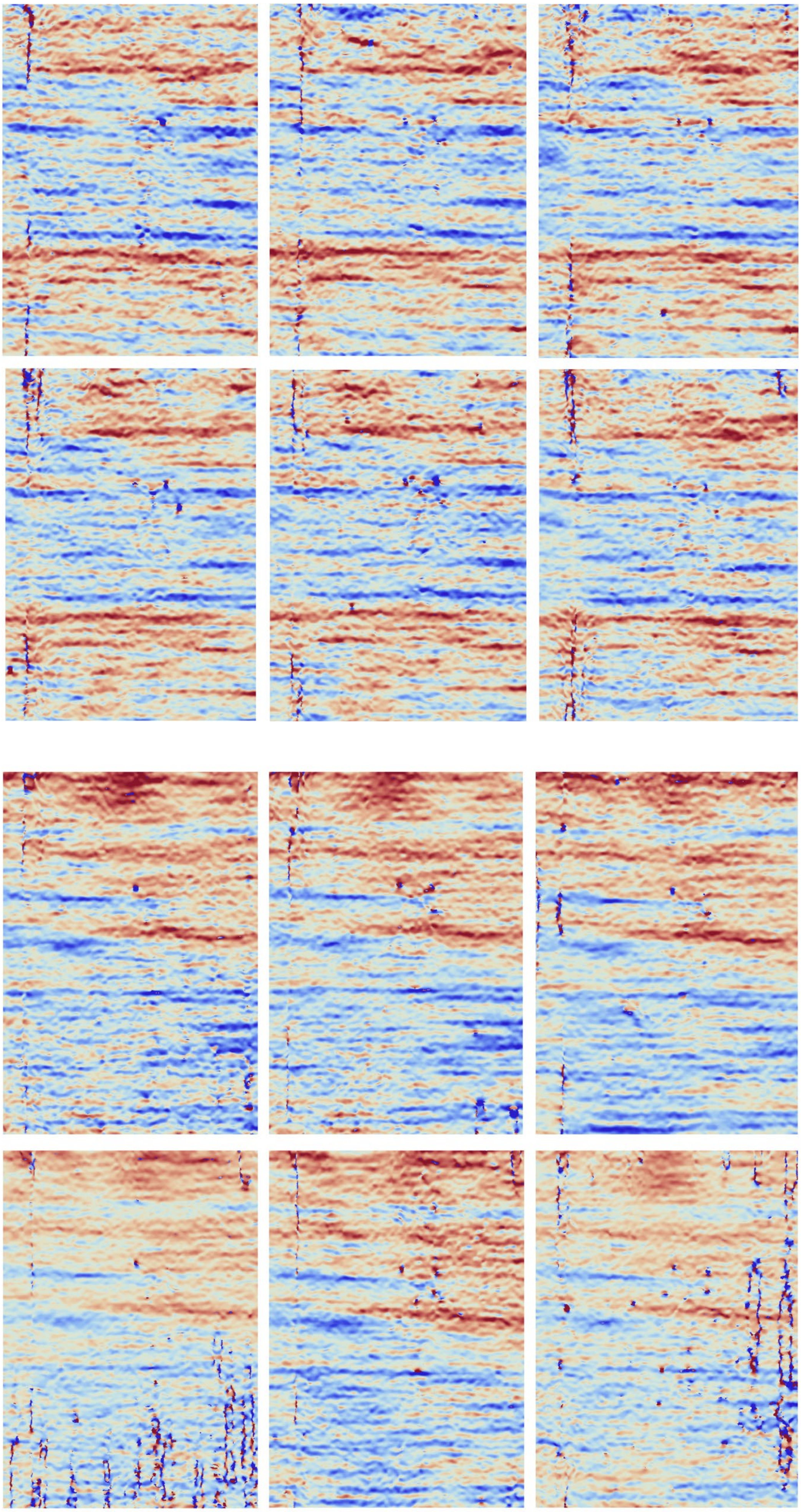

Figure 6. Laid line density maps for the watermark side of the 6 A samples (up), which all have similar patterns of high and low density regions, compared to the watermark side of the mold C samples (down), which have a visibly different pattern.

Bi-folios (1/2 sheets), quartos (1/4 sheets), and octavos (1/8 sheets) were commonly used for books or prints requiring smaller sheets of paper and often have fragmentary watermarks or countermarks. However, separated sections sharing chain lines will have the same chain angles and intervals (or ratios, if one sheet shrunk relative to the other due to differing environmental factors) at their shared edge. Thus, chain line quantification for bi-folios and quartos taken from the same half of a mold is an additional useful parameter that can be used in absentia of a watermark or for watermarks that are so fragmentary that little quantitative data can be extracted from them.

Often, however, chain line data can be inconclusive. Since a full mold, like the ones used to make the *De re militari* pages, only contained about 17 or 18 chain lines, and folios and quartos have even fewer, the resulting patterns can often appear fairly close. For example, the last four chain line intervals for the watermark side of mold D have a similar shape to the middle spacings for mold C (Figure 4). Additionally, while quartos and folios representing the same mold areas can be matched, there is no way to securely identify sheets divided parallel to the chain lines. Exemplifying this dilemma are the *De re militari* sheets, which all have similar mean intervals around 1 inch, negating that statistic as an identifying factor.

Because of the perpendicular nature of the mold wires, however, adding laid lines to the analysis presents a richer and more complex dataset which can be used to compare portions of the mold located on the same horizontal axis. The laid line method described here provides a high-resolution characterization of the density variation specific to any region on a sheet's surface. Laid line maps like those in Figure 5 demonstrate that each mold is unique across its entire two-dimensional surface and that the laid lines cannot be adequately characterized by a mean density value. As an example, while the sampled sheets from molds A and C have similar overall mean values for laid line density, the laid line maps are clearly different across the molds (Figure 6).

Further, obtaining the laid line density in two image dimensions creates a highly specific descriptor of a sheet's mold section. Using earlier analyses, the overall mean for a cropped paper might indicate potential matches, just as for full sheets, but would be inconclusive for distinguishing between papers of similar means. With laid line density mapping, however, information on the laid lines can be combined with chain line intervals to locate a paper horizontally and vertically on the mold surface to a high level of precision, thereby continually improving the data set as more moldmates are found.

A final historical note is that the molds used to produce laid paper could suffer damage or wear over time, altering the local density and angles of wires. However, the filtering effect of the Fourier transform is effective in nullifying some of the variance between sheets that may differ in this manner. For example, in the left portion of Figure 6, the sheets classified as mold C each reveal a few uniquely anomalous regions of high and low density. These variations are likely due to slight differences in the sheets' local topography, but the laid line patterns are all recognizably similar despite such noise. Additionally, in cases such as these sheets and others which comprise a single book or project, the papers were likely produced in quick succession. In consequence, only minimal wear could take place between the making of one sheet and the next, resulting in extremely similar wire patterns like the ones shown in these papers. Thus, for the purposes of specifically discovering moldmates from a narrow time frame, such as the creation of a single ream of paper, the inner structural features of samples should be even more quantitatively alike than moldmates in general.

Conclusion

As shown, any of the three methods for quantifying the structural features of handmade laid paper (watermark distances, chain line intervals, and laid line density) can be used to identify moldmates among a set; which method is used depends on the nature of the paper being studied. Among these methods, an important consideration is sheet size and whether enough of the watermark or the chain line wires are visible to use those methods for characterization. Conversely, the laid line analysis requires images or radiographs of sufficiently high resolution to capture the variance effectively. The combination of all three methods yields the most accurate description of a paper and points to the future assembly of master data sets for any mold discovered, ideally allowing the reconstruction of full-size models. These new methods would be capable of documenting self-similarity across sheets of various size and could establish the relationships of full and partial sheets to specific molds, both within a single set of papers and against potential matches from other sources. However, such an endeavor requires a renewed call for the production of many high-quality images of laid paper. Such images must have sufficient resolution to resolve the laid lines and must cover the entire surface of a documented sheet. These requirements run contrary to common practices that focus solely on recording watermarks and countermarks and neglect sheets and regions lacking those features. With enough images, the quantitative methods proposed here could assist in the creation of new datasets for active research areas, such as Rembrandt's print oeuvre, while opening new avenues into questions previously unexplored for lack of watermarks or sufficient comparable data.

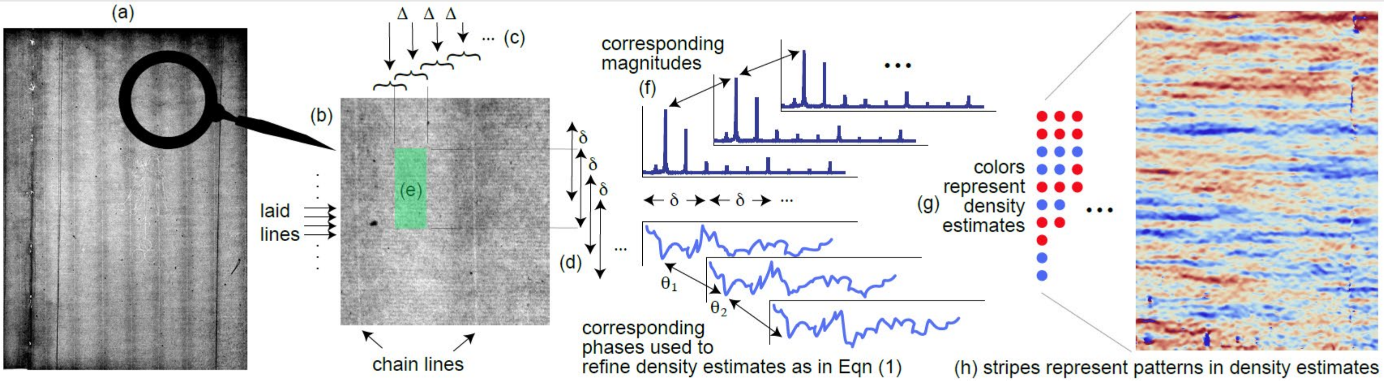

Figure 7. A visual description of the use of a 1D FFT to detect laid lines in a transmitted light image and to create density maps for the full area of the page.

Appendix I: Mathematics of the Laid Line Detection Software

A key step in van Staalduinen's laid line detection process[16], as well as the canvas thread counting processes of Johnson, et.al.[17], and Sethares[18], is locating the dominant peak in the magnitude of the two-dimensional (2D) Fourier transform. An advantage of the 2D Fourier transform is that the results are invariant to rotations in the paper. However, aligning the image vertically with the chain lines, and hence horizontally with the laid lines, is usually straightforward. This allows use of the simpler one-dimensional Fourier transform (FFT) in the algorithm used in this paper for calculating and displaying laid lines, which is diagrammed in Figure 7. A grayscale version of the transmitted light image is shown in 7(a), which contains eight vertical chain lines and a watermark of a foolscap. 7(b) shows an expanded portion of the image containing two chain lines and a large number of horizontal laid lines. A low pass filter with kernel of width Δ cm is used in 7(c) to smooth in the horizontal direction. In 7(d), FFTs of length *n* are overlapped by an amount δ (in cm). 7(e) shows a single analysis window of width Δ and height *n*.

The peak magnitudes of the FFTs are used as a rough estimate of the frequency of the laid lines and this estimate is refined using the "phase vocoder" strategy as described in Puckette and Brown[19] and as implemented in Sethares[20]. The frequency multiplied by the change in time δ must equal the change in angle, that is, $2\pi f\delta=\theta_2-\theta_1$ or some 2π multiple of this quantity. Solving for f gives

$$f_m=(\theta_2-\theta_{-1}+2\pi m)/(2\pi\delta)$$

Equation 2

for some integer *m*. The phase vocoder exploits Equation 2 by locating a pair of corresponding peaks in the magnitude spectrum of two different frames as in 7(f), and then uses the corresponding phase angles θ_1 and θ_2 to refine the estimates. The frequency estimates (in cm/laid line) are changed to density (in laid lines/cm) and then mapped into colors for visual display, where the colors represent the laid line density for each sampled point. A blue color indicates a smaller value while a red point has a higher density, as shown diagrammatically in 7(g); thus blue regions have laid lines spaced more widely while red regions have laid lines packed more tightly. Applying this same procedure to all the overlapping analysis windows across the image results in density maps such as 7(h). The stripes and colored blotches are characteristic of the individual paper molds and may provide useful information in their analysis.

Figure 5 shows laid line density maps for several of the blank sheets of De re militari, as calculated using the procedure of Figure 7. The pages were photographed at a resolution of 540 dpi (212.6 dpc). The FFT size was set to 0.8 cm (170 pixels) and the delta parameters were $\delta=\Delta=0.05$ cm (10 pixels).

Appendix II: Additional Figures and Data

Similar to Figure 4, which shows the ratio patterns and tolerances for the watermark sides of the four molds, Figure 8 displays the calculated trends for the countermark sides of the molds.

Figure 6 displays the laid line density maps for six samples each of the watermark side of molds A and C; those maps are reproduced here (Figures 9-16) in larger scale along with the density maps for all the sampled countermark and watermark sides of each mold. Each image is accompanied by a table (Tables 3-10) similar to Table 2 describing the mean and standard deviation of the density across the entire sheet, further demonstrating the similarity between moldmates.

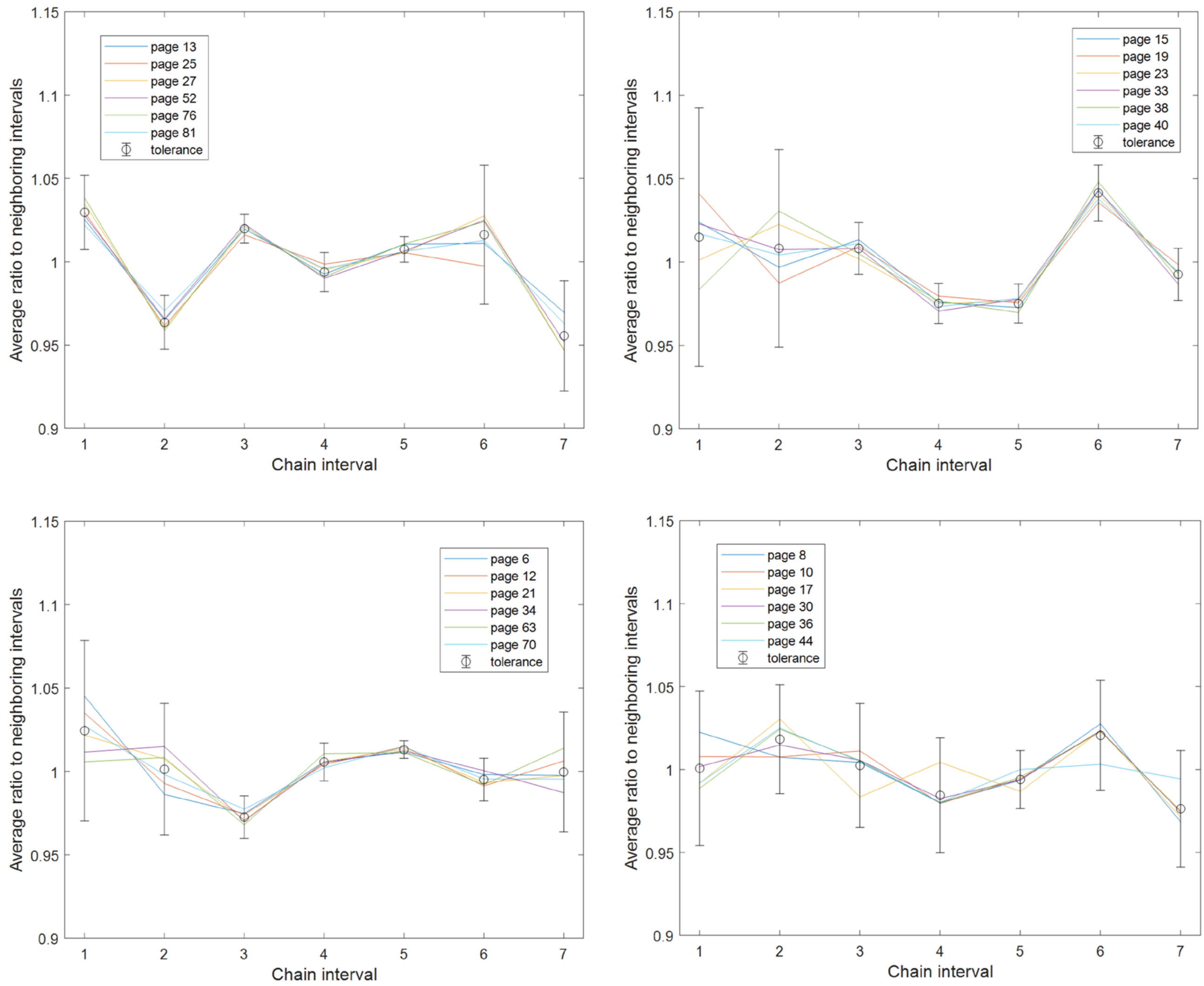

Figure 8. Spaces between chain lines for the countermark side of the four molds, with tolerance bars. From left to right, top to bottom: molds A, B, C, and D.

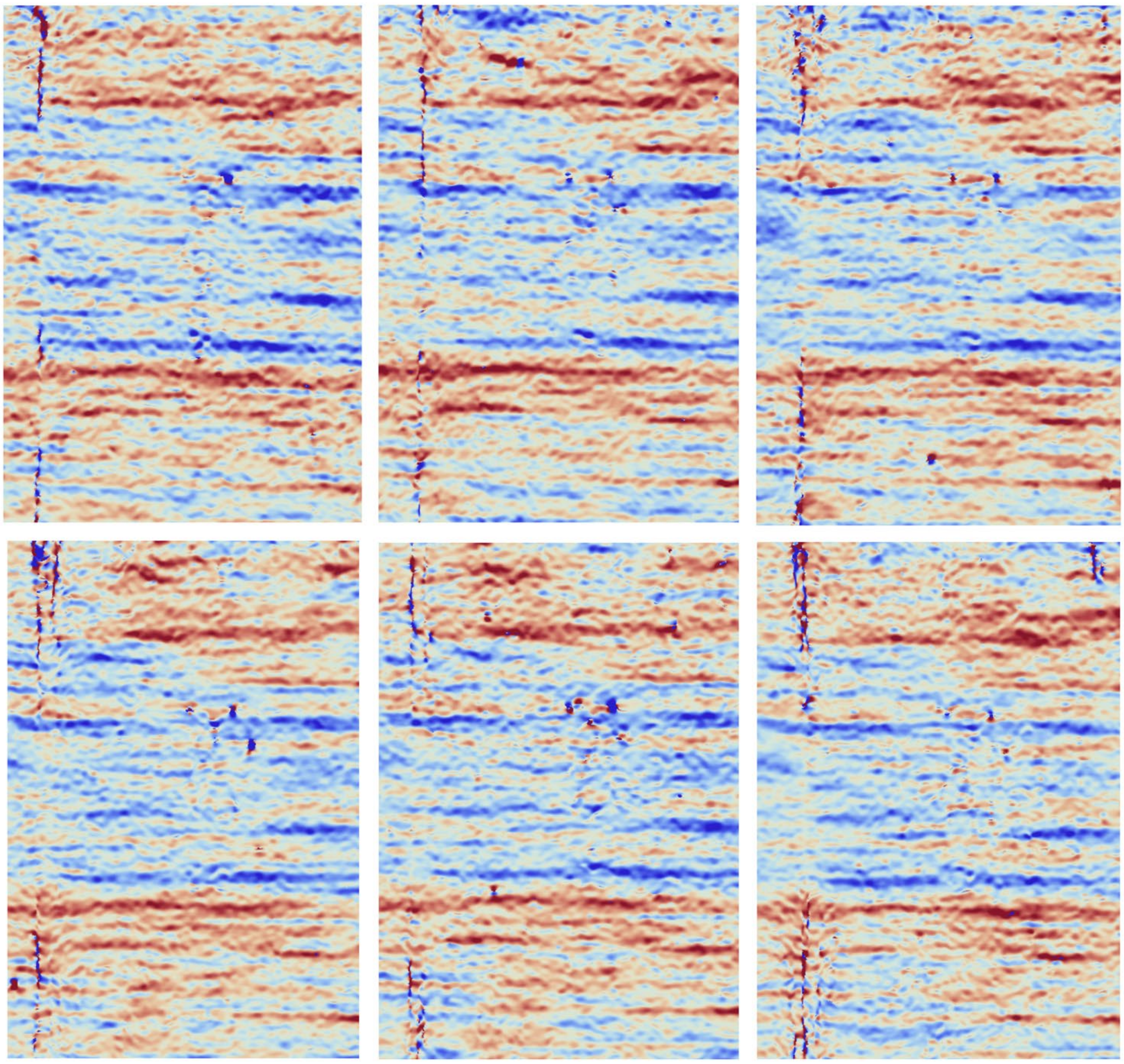

Figure 9. Laid line density maps for the watermark side of the 6 sampled sheets from mold A.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
14	10.652	0.294
26	10.636	0.295
28	10.603	0.289
51	10.632	0.304
75	10.636	0.300
82	10.600	0.306

Table 3. Mean and standard deviation of laid line density for the watermark side of the 6 sampled sheets from mold A.

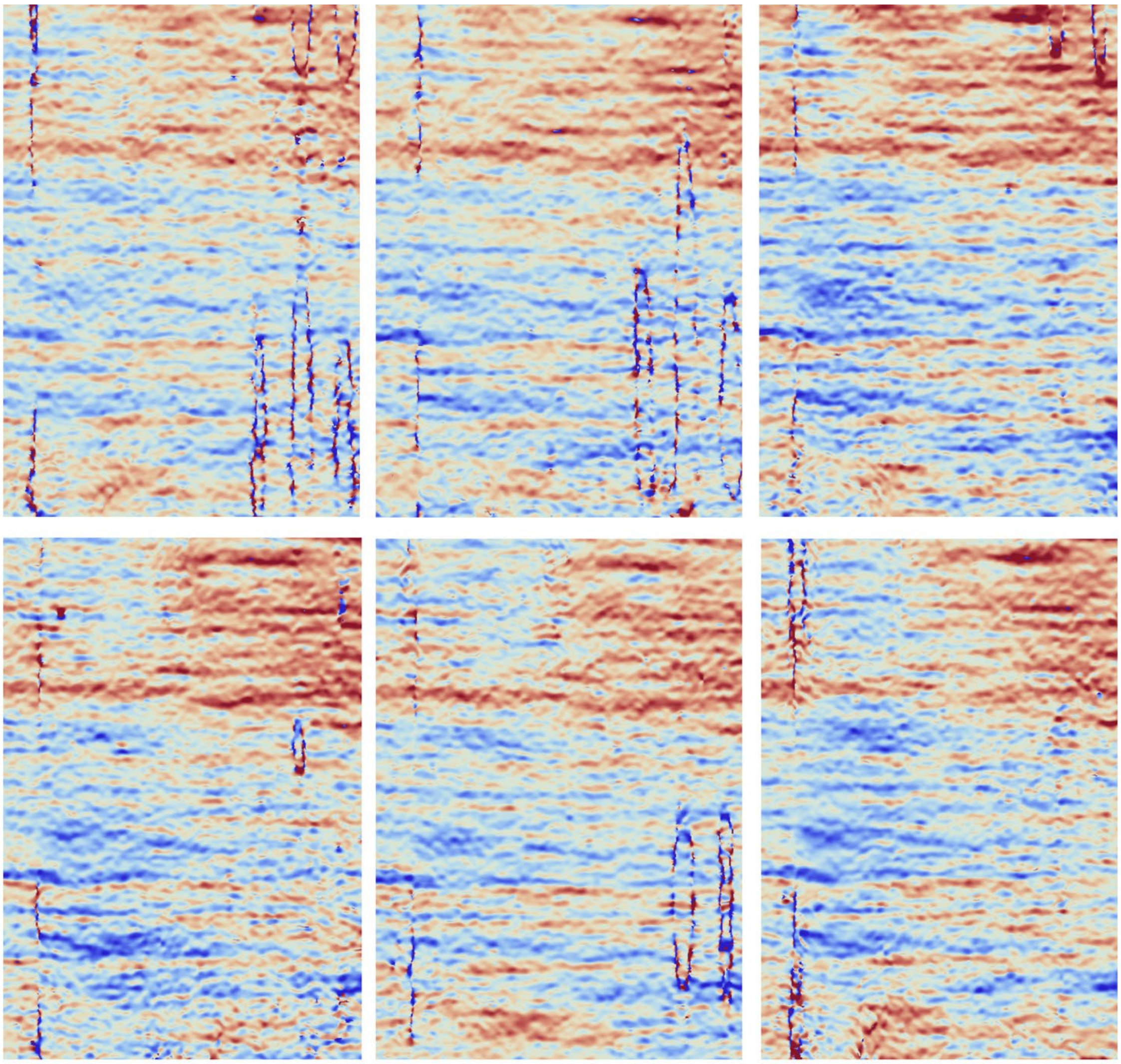

Figure 10. Laid line density maps for the countermark side of the 6 sampled sheets from mold A.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
13	10.633	0.372
25	10.684	0.356
27	10.628	0.282
52	10.618	0.311
76	10.609	0.315
81	10.586	0.301

Table 4. Mean and standard deviation of laid line density for the countermark side of the 6 sampled sheets from mold A.

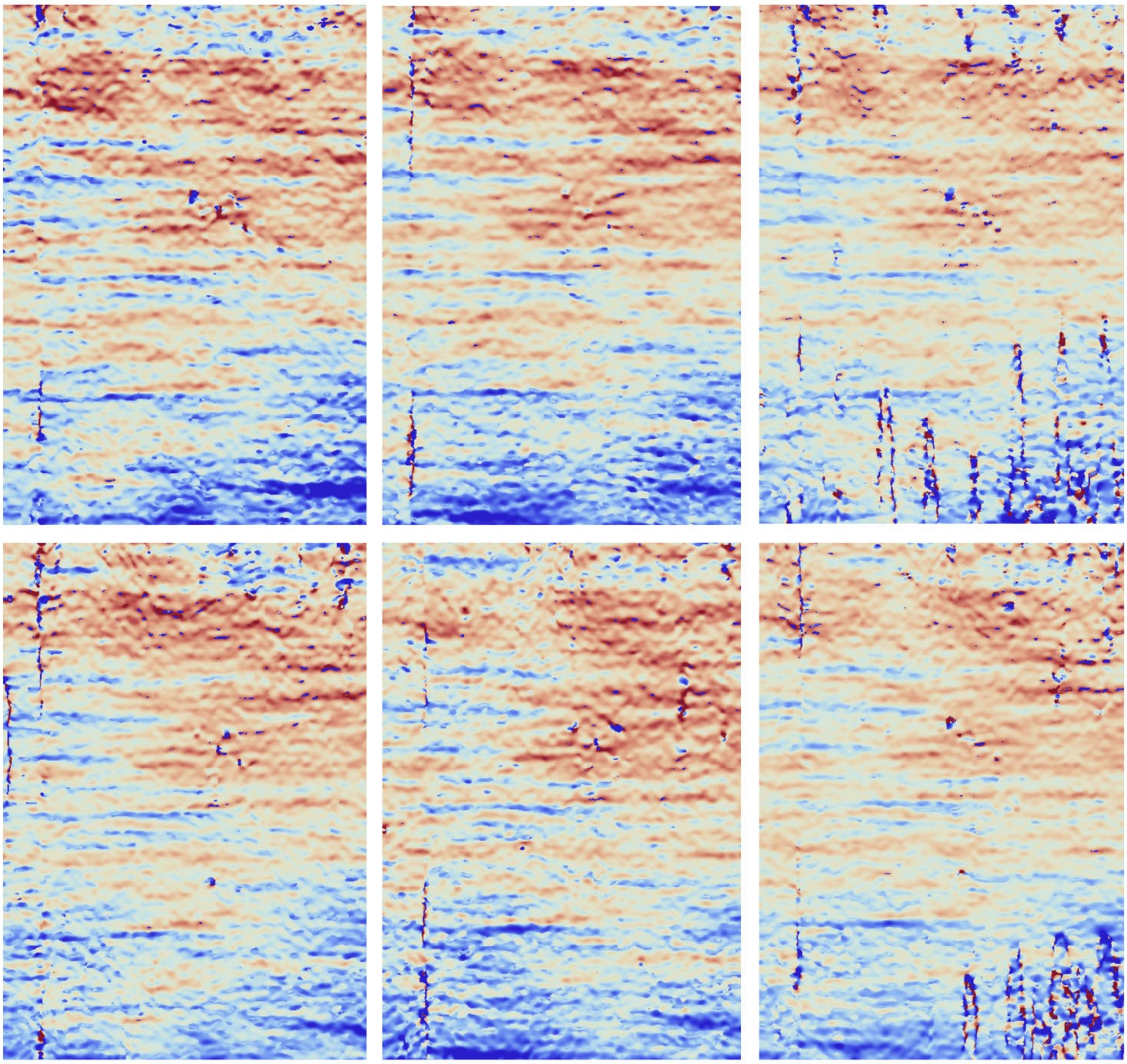

Figure 11. Laid line density maps for the watermark side of the 6 sampled sheets from mold B.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
16	11.018	0.307
20	11.023	0.349
24	11.032	0.452
32	11.008	0.363
39	10.927	0.337
41	10.947	0.432

Table 5. Mean and standard deviation of laid line density for the watermark side of the 6 sampled sheets from mold B.

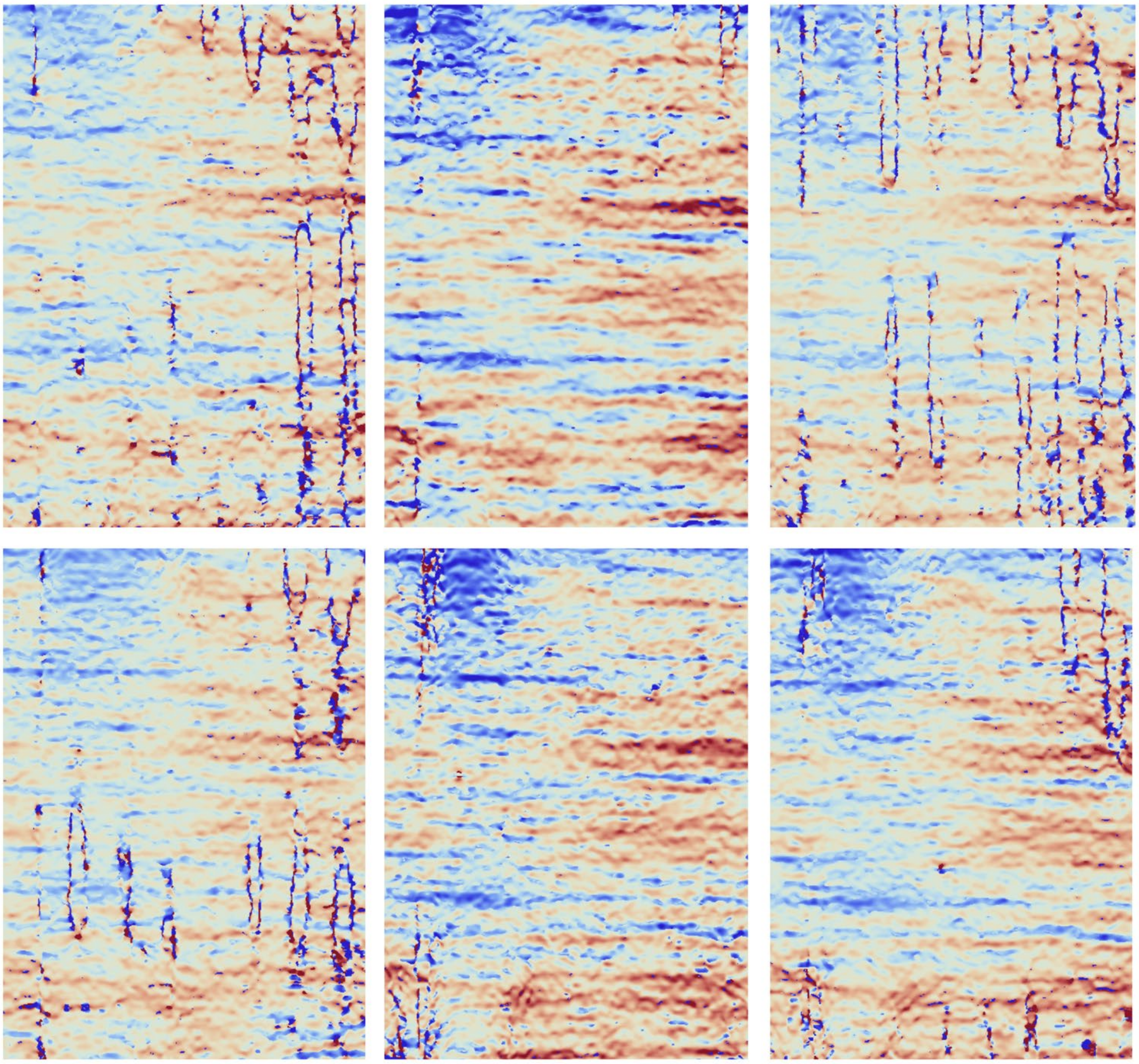

Figure 12. Laid line density maps for the countermark side of the 6 sampled sheets from mold B.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
15	11.036	0.452
19	11.054	0.298
23	10.991	0.497
33	10.964	0.474
38	10.908	0.330
40	10.994	0.381

Table 6. Mean and standard deviation of laid line density for the countermark side of the 6 sampled sheets from mold B.

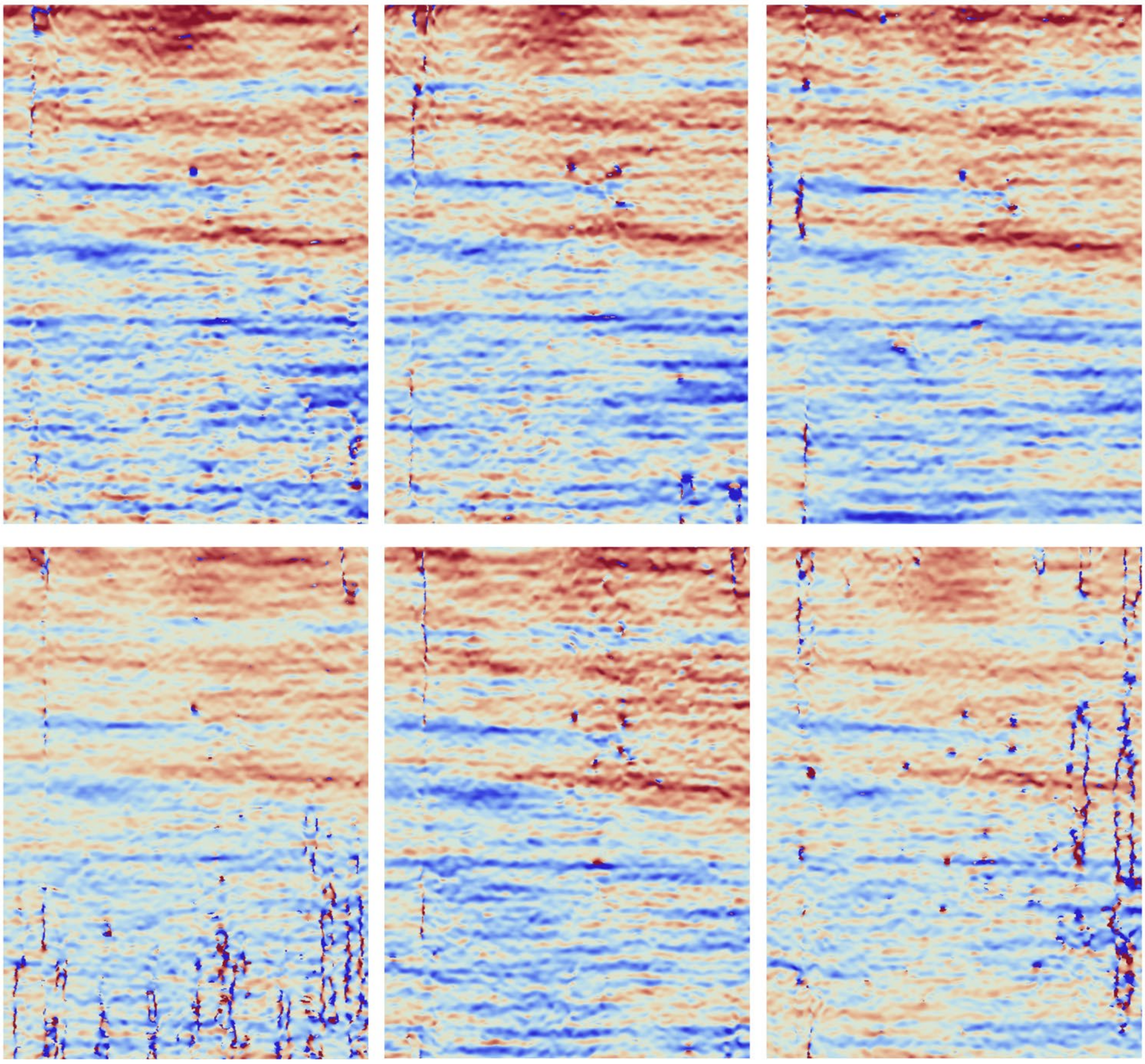

Figure 13. Laid line density maps for the watermark side of the 6 sampled sheets from mold C.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
5	10.682	0.337
11	10.611	0.342
22	10.621	0.348
35	10.645	0.481
64	10.641	0.350
69	10.646	0.460

Table 7. Mean and standard deviation of laid line density for the watermark side of the 6 sampled sheets from mold C.

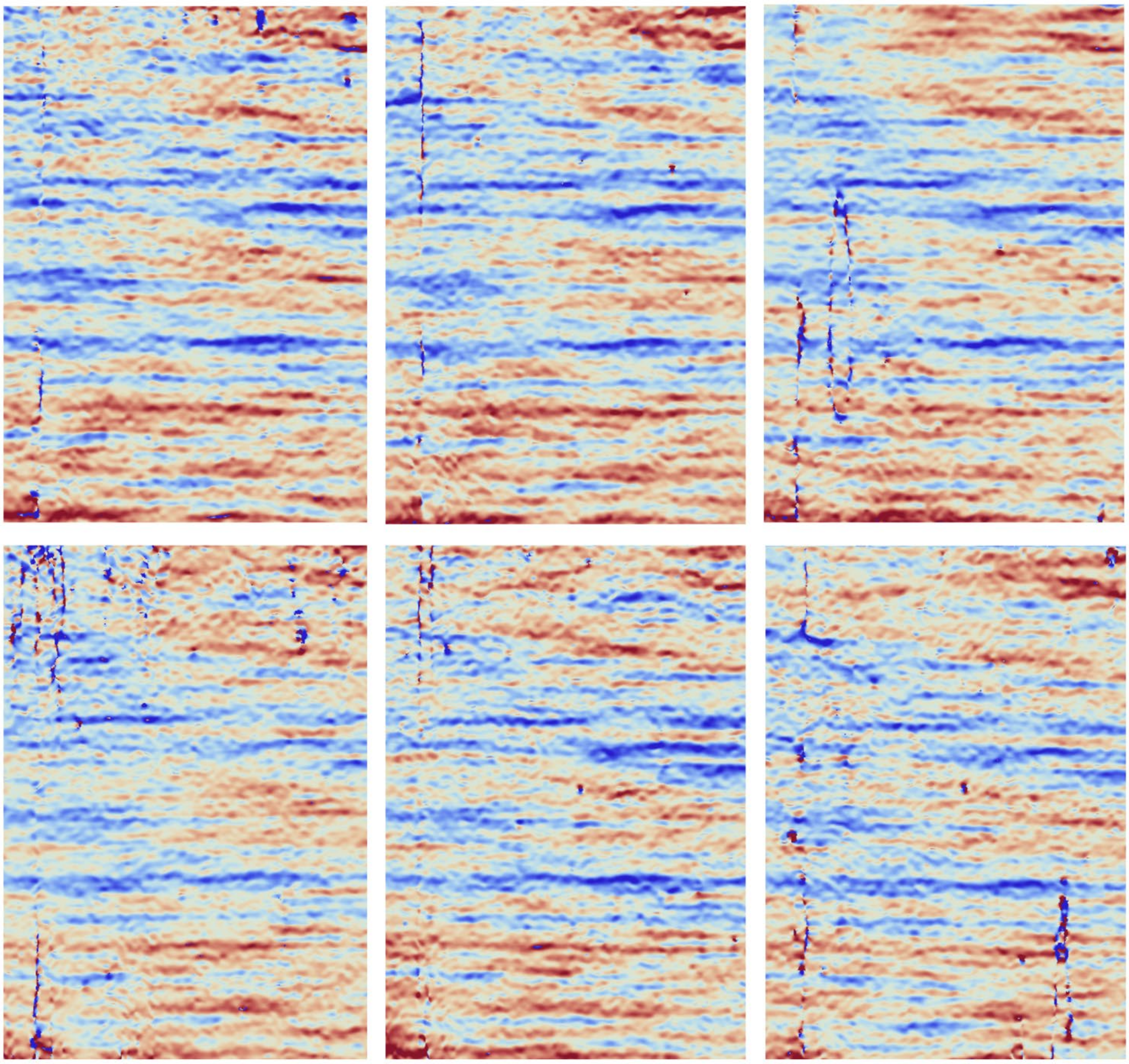

Figure 14. Laid line density maps for the countermark side of the 6 sampled sheets from mold C.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
6	10.672	0.311
12	10.610	0.315
21	10.643	0.333
34	10.663	0.375
63	10.653	0.312
70	10.625	0.336

Table 8. Mean and standard deviation of laid line density for the countermark side of the 6 sampled sheets from mold C.

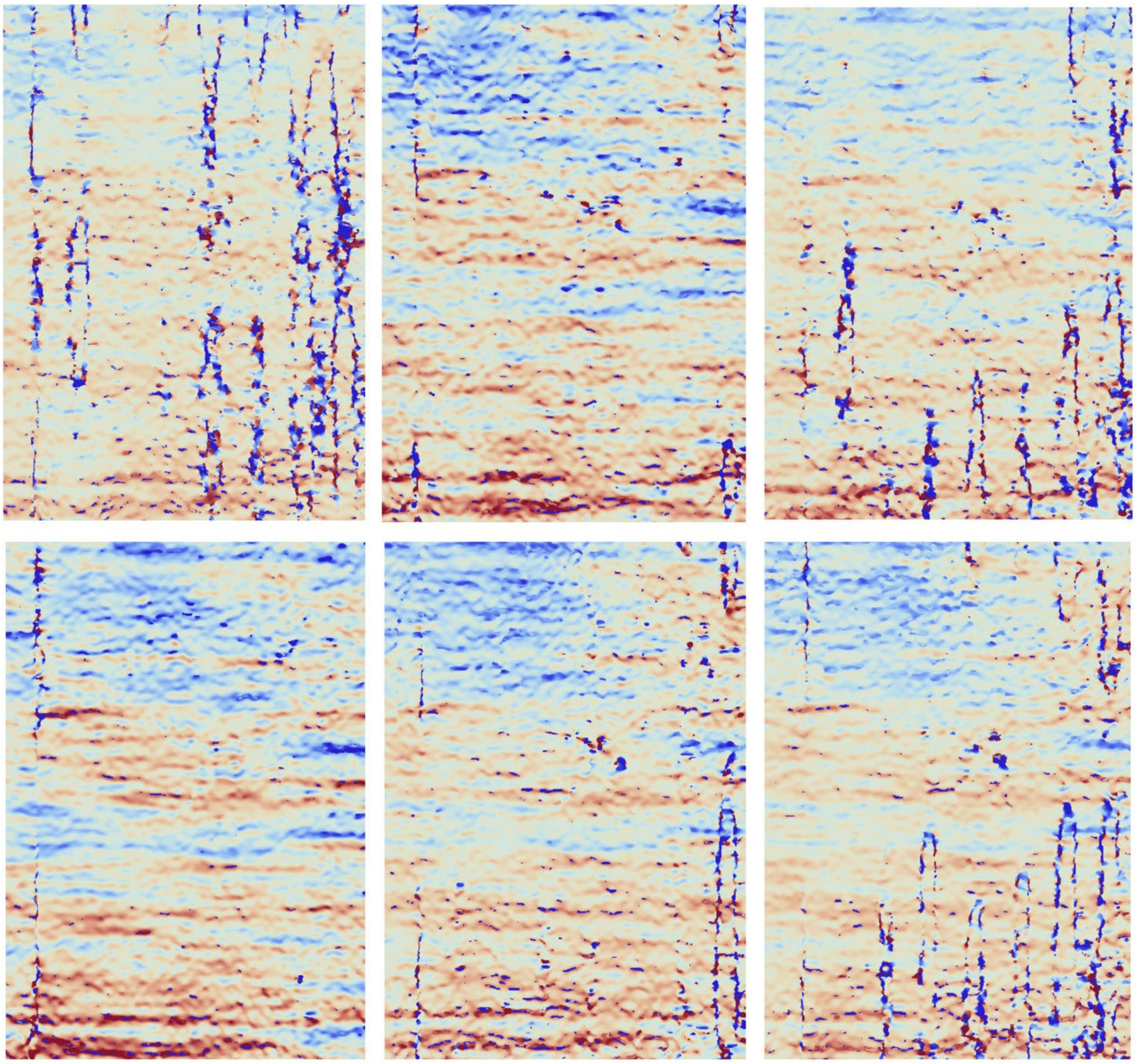

Figure 15. Laid line density maps for the watermark side of the 6 sampled sheets from mold D.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
7	11.080	0.668
9	11.165	0.346
18	11.109	0.509
29	11.174	0.301
37	11.182	0.423
45	11.133	0.507

Table 9. Mean and standard deviation of laid line density for the watermark side of the 6 sampled sheets from mold D.

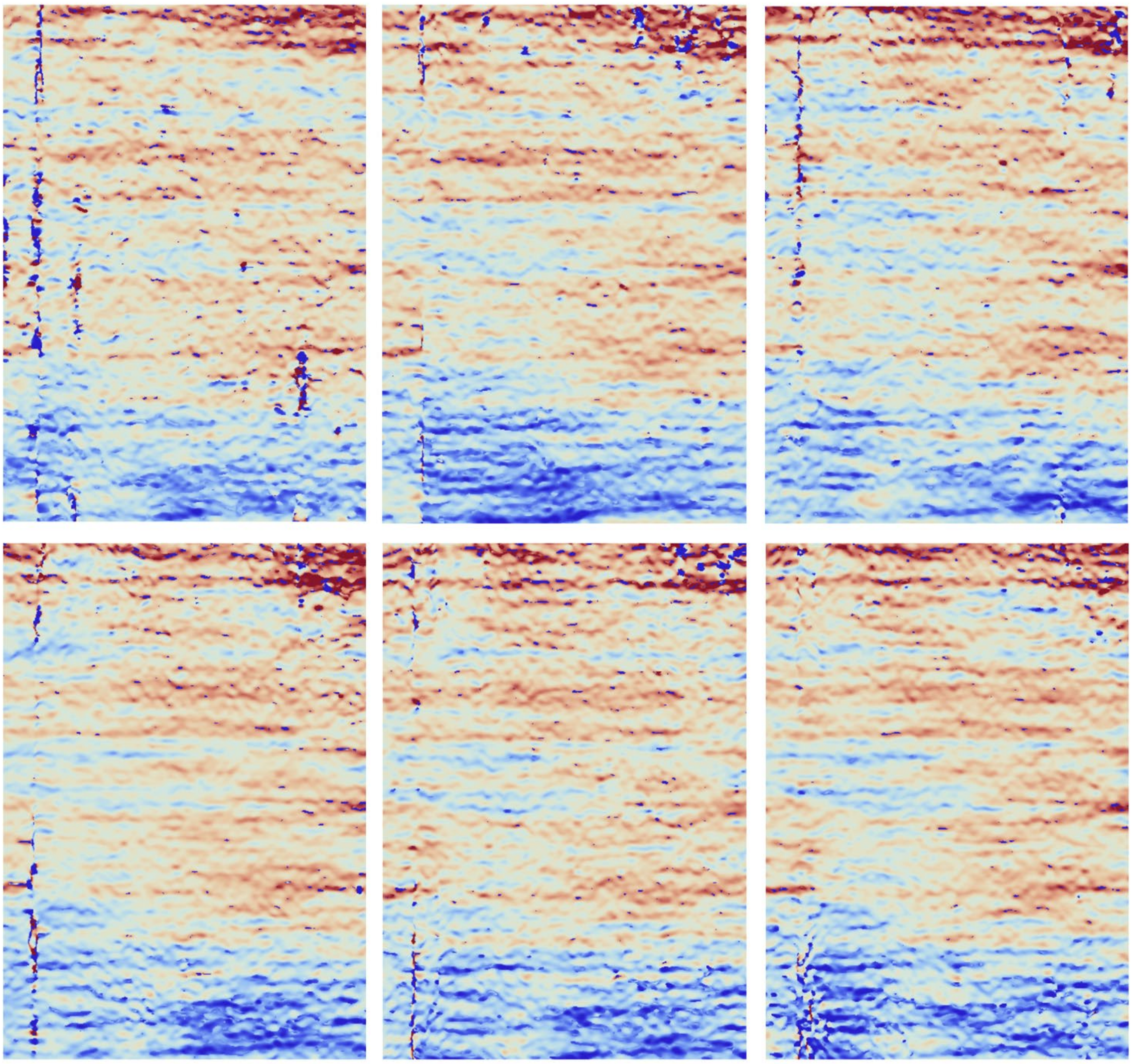

Figure 16. Laid line density maps for the countermark side of the 6 sampled sheets from mold D.

PAGE NUMBER	MEAN DENSITY (LINES/CM)	STANDARD DEVIATION (LINES/CM)
8	11.150	0.413
10	11.134	0.362
17	11.153	0.353
30	11.168	0.353
36	11.141	0.340
44	11.129	0.331

Table 10. Mean and standard deviation of laid line density for the countermark side of the 6 sampled sheets from mold D.

NOTES

[1] Harris, Neil. “A History of Handmade Paper: The Basic Problem.” In *Paper and Watermarks as Bibliographical Evidence*, 2nd ed. Lyon: L’Institute D’Histoire du Livre, 2017. PDF.
[2] Hansen, Lara D. “Between the Sheets: The Use of Paper Evidence in Bibliographical Reconstruction of the Shakespeare First Folio.” Ph.D. Thesis, University of Nevada, Reno, 2014.
[3] Hinterding, Erik. “Rembrandt as etcher: The practice of production and distribution.” In *Sound & Vision*, vol. 1. Ouderkerk aan den Ijssel: Sound & Vision Publishers, 2006.
[4] Johnson, Jr., C. Richard. “Decision Trees in Watermark Identification in Rembrandt’s Etchings.” *Journal of Historians of Netherlandish Art* 12, no. 2 (2020).
[5] Doynov, Plamen. “Framework for Automatic Identification of Paper Watermarks with Chain Codes.” Ph.D. Thesis, University of Missouri – Kansas City, 2017.
[6] Harris, Neil. “The Shape of Paper.” In *Paper and Watermarks as Bibliographical Evidence*, 2nd ed. Lyon: L’Institute D’Histoire du Livre, 2017. PDF.
[7] Ellis, Margaret Holben and C. Richard Johnson, Jr. “Computational Connoisseurship: Enhanced examination using automated image analysis.” *Visual Resources* 35, no.1/2 (2019): 125-140.
[8] Frank, Emily B., Margaret Holben Ellis, Lydia Aikenhead, and Paul Messier. “The computational analysis of watermarks: Setting the stage for the development of a Watermark Imaging Bo (WImBo).” IEEE 2018 *52nd Annual Conference on Information Sciences and Systems* (2018): 1-4.
[9] Xi, Xuelei, C. Richard Johnson, Jr., David Conathan, William A. Sethares, and Amanda House. “Automated chain line marking and pattern matching in radiographs of Rembrandt’s prints.” *2016 50th Asilomar Conference on Signals, Systems and Computers* (2016): 134-138.
[10] van Staalduinen, Mark, Jan C.A. van der Lubbe, Eric Backer, and Pavel Paclik. “Paper retrieval based on specific paper features: Chain and laid lines.” *Multimedia Content Representation, Classification, and Security: Lecture Notes in Computer Science*, 4105 (2006): 346-353.
[11] Johnson, Jr., C. Richard. “Decision Trees in Watermark Identification in Rembrandt’s Etchings.” *Journal of Historians of Netherlandish Art* 12, no. 2 (2020).
[12] Xi, Xuelei, C. Richard Johnson, Jr., David Conathan, William A. Sethares, and Amanda House. “Automated chain line marking and pattern matching in radiographs of Rembrandt’s prints.” *2016 50th Asilomar Conference on Signals, Systems and Computers* (2016): 134-138.
[13] van Staalduinen, Mark. “Content-Based Paper Retrieval: Towards Reconstruction of Art History.” Ph.D. Thesis, University of Delft, 2010.
[14] Johnson, Don H., C. Richard Johnson, Jr., and Robert G. Erdmann. “Weave Analysis of Paintings on Canvas from Radiographs.” *Signal Processing* (Special Issue on Image Processing for Art Investigation) 93 (2013): 527-540.
[15] Sethares, William A. “Automated Creation of Weave Maps.” In *Counting Vermeer*, edited by C. Richard Johnson, Jr. and William A. Sethares. The Hague: RKD Monograph, 2018. http://countingvermeer.rkdmonographs.nl/chapter-5-automated-creation-of-weave-maps
[16] van Staalduinen, Mark. “Content-Based Paper Retrieval: Towards Reconstruction of Art History.” Ph.D. Thesis, University of Delft, 2010.
[17] Johnson, Don H., C. Richard Johnson, Jr., and Robert G. Erdmann. “Weave Analysis of Paintings on Canvas from Radiographs.” *Signal Processing* (Special Issue on Image Processing for Art Investigation) 93 (2013): 527-540.
[18] Sethares, William A. “Automated Creation of Weave Maps.” In Counting Vermeer, edited by C. Richard Johnson, Jr. and William A. Sethares. The Hague: RKD Monograph, 2018. http://countingvermeer.rkdmonographs.nl/chapter-5-automated-creation-of-weave-maps
[19] Puckette, Miller S. and Judith C. Brown. “Accuracy of frequency estimates using the phase vocoder.” IEEE Transactions on Speech and Audio Processing 6, issue 2 (1998): 166-176.
[20] Sethares, William A. *Rhythm and Transforms*. Berlin; London: Springer Verlag, 2007.

ACKNOWLEDGEMENTS

S.F. Gorske and C.R. Johnson, Jr. were supported by National Science Foundation Grant CCF-1822007 and Getty Foundation Grant ORG-201943572. W.A. Sethares was supported by Getty Foundation Grant ORG-201943572. The authors declare no competing financial interests related to this work.

SARA F. GORSKE studied Materials Science and Engineering and the History of Art at Cornell University (B.S. '20). Her interests in the intersection of art and engineering led her to the research group of Dr. C. Richard Johnson, Jr. in the Department of Electrical and Computer Engineering at Cornell University in the spring of 2018. During her time in the group, she contributed to the Watermarks in Rembrandt's Etchings (WIRE) project to help codify the identification of watermarks through decision trees and assisted in the development of semi-automatic and automatic chain line and laid line marking programs by exploiting the signals in images of backlit paper.

Correspondence e-mail: sg742@cornell.edu

C. RICHARD JOHNSON, JR. is the Geoffrey S. M. Hedrick Senior Professor of Engineering at Cornell University in Ithaca, NY. He received the first PhD minor in Art History granted by Stanford University in 1977, along with a PhD in Electrical Engineering. Since 2007, his research has used digital signal processing to address issues in computational art history, primarily through the matching of manufactured patterns in art supports. Professor Johnson's contributions to canvas weave-mapping using Fourier analysis for rollmate identification, photographic paper classification via texture analysis, laid paper chain line pattern analysis for moldmate verification, and watermark identification using decision-tree construction for batchmate designation have appeared in the Burlington Magazine, the Metropolitan Museum Journal, Studies in Conservation, the Journal of the American Institute for Conservation, Art Matters, the Journal of Historians of Netherlandish Art, the International Journal for Digital Art History, the Journal of Interactive Technology and Pedagogy, Visual Resources, Signal Processing, and the IEEE Signal Processing Magazine. He is the lead editor of and a contributing author to Counting Vermeer: Using Weave Maps to Study Vermeer's Canvases, RKD Studies, 2017 (http://countingvermeer.rkdmonographs.nl).

Correspondence e-mail: crj2@cornell.edu

WILLIAM A. SETHARES received the Ph.D. degree in electrical engineering from Cornell University, and is currently Professor in the Department of Electrical and Computer Engineering at the University of Wisconsin in Madison. Prof Sethares has held visiting positions at the Australian National University, at the Technical Institute in Gdansk, and at the NASA Ames Research Center. He is a scientific researcher at the Rijksmuseum in Amsterdam and is the Honorary International Chair Professor at the National Taipei University of Technology. His research interests include adaptation and learning in image and signal processing with special focus on the imaging and the interpretation of historical papers and canvases. Dr. Sethares is the author of five books, including Tuning, Timbre, Spectrum, Scale, and is co-author of a recent monograph (Counting Vermeer) which examines all the extant canvases of Johannes Vermeer.

Correspondence e-mail: sethares@wisc.edu

MARGARET HOLBEN ELLIS received her M.A. in Art History and Advanced Certificate in Conservation, Institute of Fine Arts, NYU and is currently the Eugene Thaw Professor of Paper Conservation at the Institute of Fine Arts, also serving as Director of the Thaw Conservation Center at the Morgan Library & Museum. She has published and lectured on technical issues related to artists ranging from Raphael and Titian to Pollock, Samaras, Dubuffet and Lichtenstein with her research interests in artists' materials similarly far-ranging. At the moment she is exploring the significance of "paper as part of the picture", the possibility of applying computer-generated pattern recognition algorithms to the characterization of antique papers, specifically as used in Rembrandt prints, and Jean Dubuffet's assemblages d'empreintes. She served as volume editor for Historical and Philosophical Issues in the Conservation of Works of Art on Paper, which was published by the Getty Conservation Institute in 2014. Her revised edition of The Care of Prints and Drawings was published in 2016.

Correspondence e-mail: mhe1@nyu.edu

PAUL MESSIER is the Pritzker Director of the Lens Media Lab and Chair of the Institute for the Preservation of Cultural Heritage, both at Yale University. The IPCH is a research collaborative, dedicated to the preservation and interpretation of material culture. Within IPCH, the LML focuses on collection-scale analysis and interpretation of artist materials. Paul has published widely on innovative methods characterize and classify works of art and is the 2018 recipient of the Award for Distinction in Scholarship and Conservation jointly presented by the College Art Association and the American Institute for Conservation.

Correspondence e-mail: paul.messier@yale.edu