

Measurement

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Measurement is a cultural technique which aims at a numerical description of facts. As such it is fundamental for the use of mathematical techniques of representation and deduction. The history of the progressive formation of quantitative concepts in the expansion of the mathematical sciences proves to be a complicated process. This process occurs in a space spanned by the three dimensions of existing knowledge, technology, and symbolic resources and eventually results in a match between manipulations on a symbolic and an instrumental level. While the success of the mathematical natural sciences gave rise to Platonic speculations about a mathematical constitution of reality itself, the aspect of instrumental mediation in the process of measurement points toward a modern concept of nature that views it in terms of technical control, thus inscribing measurement into the modern project of a technical mastery of nature.

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Preliminary note

The philosophical ‘measurement problem’ in a general sense deals with measurement as a precondition of the mathematical representation and processing of facts. In discussions of quantum mechanics and its interpretation a ‘measurement problem’ also shows up, pointing to the problem that on the quantum level the interaction between system and measuring instrument has to be understood in a completely new way (Neumann 1932). From a quantum mechanical point of view, the interaction with the measuring instrument not only represents a possible disturbance of the state of the system (as when, for example, a thermometer influences the temperature of a liquid) but shifts the whole system from an indeterminate into a determinate state (the so-called “collapse of the wave function”). However, whether a determinate state can be ascribed to a (quantum mechanical) system independently of the measurement, and how it is possible to describe states in the language of numbers at all, are two different questions that can be treated independently of each other. The present lemma only deals with the latter of these two questions.

1. Measuring and its Meaning

Measurement is a cultural technique closely related to counting. In the Western tradition it can be traced back to the first advanced civilizations in Mesopotamia and Egypt. Initially of relevance primarily in the economic and technical spheres, it later came to play a crucial role in the modern natural sciences. In accordance with its enormous cultural significance, measurement is the subject of an extensive but also very heterogeneous literature. One finds side by side (1.) the genre of practical guides to measurement, which, however, merges into a theoretical-scientific literature, e.g. from the art of field measurement to geometry, from the art of weighing to statics; (2.) a modern metrological literature dealing with both practical and theoretical questions of measurement and, in particular, the construction of units and systems of units; (3.) the formal theory of scales of measurement, which theoretically deals with the construction and classification of scale types; (4.) measurement theory in the sense of philosophy of science; and (5.) finally, a historical-sociological literature on measurement, its history and its social context.

According to a famous definition by the American psychologist Stanley S. Stevens, which paved the way for the formal theory of measurement and became decisive for the second half of the twentieth century and the theory of science which established itself during this period, measurement “in the broadest sense” can be understood as “the assignment of numerals to objects or events according to rules” (Stevens 1946: 677). Through measurement, a conceptually fixed property can be determined and expressed numerically in its quantitative extension or intensity. In practical contexts the numerical expression is sought because of its specific information content. In scientific-theoretical contexts a new aspect is added: numerical expressions allow us to apply a mathematical approach (understood as a special technique of deduction) and thus to formulate mathematical laws of nature and to calculate predictions from current measurements and laws of nature.

In counting, which is more fundamental from a logical point of view, conventional, standardized numerals or words (starting with “one”) are successively assigned to conceptually fixed and individually identifiable objects of a concrete or abstract nature (“apples”, “neutrinos”; in a ‘reflexive’ second order application, numbers themselves can be counted too, etc.; cf. Janovskaja [1936] 2011). This assignment allows us to apply relations of equality (“same number”) and order (“more”, “less”) to pairs of sets of objects. In measurement, we are dealing with a conceptually fixed property class formed from a gradable property, i.e. a property that can vary in its size, amount, degree or intensity and can thus be expressed in a numerical way. This numerical determination takes place ‘beneath’ the level of qualitative predicators. Take colour as an example. In order to use colour terms such as “blue”, “green”, “yellow”, “red”, etc., the colour spectrum is cut into sections in a ‘conventional’ way (e.g. in a way determined by the sensory physiology of the human eye, by practical needs, cultural conventions, etc.). Numerical terms such as “430 nanometer wavelength” can take the place of such predicators but they work with a ‘finer granularity’. When, for example, we distinguish within ultramarine blue light of the wavelengths of 430 nanometers and of 435 nanometers, we identify a difference which can no longer be grasped through qualitative predictors. Bertrand Russell therefore spoke

of quantitative terms as “a conception of difference without a difference of conception” (Russell 1897: 340). Put in other words: a difference in size is a difference between two things that need not show any difference in their purely qualitative description. Christian Wolff defined “size” (*quantitas*) in the same sense as the “discrimen internum similitudinis”, i.e. the inner difference of the similar, or as that “quo similia salva similitudine intrinse differre possunt”, i.e. that in which similar things can differ without losing their similarity (Wolff [1763] 1962: 273). With the use of rational and real numbers, numerical determination is possible - at least theoretically - to any degree of accuracy, although practical measurement is always subject to an inaccuracy that is itself expressed quantitatively as “measurement error” or “measurement uncertainty”.

In order numerically to determine a gradable property by measurement, it is necessary to define a unit (formally or informally). Historically, the role of the unit is often played by natural objects or objects of practical importance (“foot” and “cubit” – from Latin “cubitus”, “elbow” or “forearm” – in length measurement, “bushel” as a unit of volume, etc.). The choice of the unit may also depend on the context, which is still the case in science (e.g. the use of light years instead of meters as astronomical measures of length or the atomic unit of mass u instead of kilograms in chemistry and atomic physics). Today’s basic units of the International System of Units (SI) are coupled to natural constants and thus defined in a highly theory-laden way (Courtenay et al. 2019). Once the unit has been determined, measuring is reduced to a counting process: it is now counted how often the unit “fits into” the quantity to be determined (in a sense specified below).

Even if, from a logical perspective, counting is more basic than measuring, we must not deduce from this that, in historical terms, counting preceded measurement. Rudimentary counting techniques of a verbal and nonverbal but prescriptive nature probably existed before the first advanced civilizations and before the Neolithic revolution (see e.g. Overmann 2014; d’Errico et al. 2017). Written numerals, which in fact seem to be the oldest characters as such, only emerged in a historical context in which measurements were already being taken, as documented by the occurrence of signs for units of measurement (Nissen et al. 1993). Historically, it would therefore probably be misleading to regard

measurement as the application of already existing numerical signs in a new, derived technique. Writing, systems of numerals and units of measurement seem instead to have emerged simultaneously and to have influenced each other in their development.

2. From Measurement to Measurement Theory

Reflection on measurement, especially on the conditions of the possibility of measurement and on its limits, appeared much later, accompanying the extension of measurement to new areas in the course of the creation and development of the modern sciences. Following a suggestion of Gernot Böhme (1976a), one can distinguish between “quantification” as the formation of concepts that give a quantitative structure to the phenomena, and (as a second, logically independent step) “metrication” as the mapping of these empirical structures (the “empirical relational systems”) onto numbers through the construction of a metric. The reflection on measurement seems to have been prompted above all by the difficulties of quantification of new phenomena. Examples include Gottfried Wilhelm Leibniz’ reflections on the “measure of the moving force” of 1696, Thomas Reid’s criticism of quantitative moral philosophy of 1748, and the extensive controversy on “psychophysics” that was triggered in the 19th century by Theodor Gustav Fechner’s attempt to measure the strength of sensations (see the materials in Schlaudt 2009a, and on psychophysics see in particular Heidelberger 2004). A consensus quickly crystallized in this literature that a necessary condition of measurement is to find empirical counterparts to arithmetical operations and relations (especially addition and equality), thus linking the above-mentioned idea of “fitting” (a multiple or submultiple of) the unit and the quantity to be measured with a concrete operational instruction (Helmholtz 1887). Formal measurement theory, which is oriented towards psychological research and which dominated in the 20th century, was primarily interested in the construction of different types of scales (Krantz et al. 1971–1990). The role of measuring instruments, which was still present, for example, in the work of Ernst Mach (1896) or the young Rudolf Carnap (1926), has only come back into focus in recent years in the context of the “practical turn” (Chang 2004; Tal 2013). So-called “protophysics” seems to have been the only approach

in philosophy of science which demanded that the theory of science begin with a systematic reconstruction of measurement technology (Lorenzen 1987; Janich 1997). Parallel to formal measurement theory, a separate discipline dealing with practical and theoretical problems of measurement has developed under the name of metrology. Part of its effort is devoted to the development of a conceptual framework of metrology in the *Vocabulaire international de métrologie* (JCGM 2012), which has, however, a bias towards physics. In the philosophy of science, measurement raises two questions: firstly, can the reliability of measurement procedures be justified, and to what extent does this show the measurement process to be dependent on theoretical knowledge (“theory-laden”)? Particularly interesting is the case of indirect measurement, determining one quantity through the measurement of another one, as for example in temperature measurement by measuring the (temperature-dependent) volume of mercury or a thermoelectric potential (Mach 1896; Chang 2004). The second question addresses the classical problem of the relationship between representation and reality with the typical poles of realistic and constructivist positions (see Ellis 1968; Falkenburg 1997; van Fraassen 2008).

From the perspective of a historical epistemology, quantification can be described as an attempt to establish a structural correspondence between two techniques, namely operating with measuring instruments and units on the one hand and the symbolic technique of operating with numbers and algebraic quantities on the other. Both sides, i.e. not only the technique of measurement but also that of mathematics, are historically changing and developing. Numbers only gradually emerged (Damerow 1994) and gradually expanded (Neal 2002), and algebraic quantity calculi developed from the ancient and early modern notation of proportions to modern algebra, first of scalar quantities but then also of more complex entities such as vectors (directed quantities) and tensors (van der Waerden 1985).

In this perspective, the emergence of quantitative concepts presents itself as a complicated process. While the natural sciences were initially able to draw on a preexisting, everyday measuring technique, other sciences striving for mathematization, such as chemistry or psychology, had to create completely new

quantitative terms from scratch. But physics, too, soon had to introduce new concepts to cope with the phenomena. A first example can already be found in the ancient theory of the lever in Aristotle (*Mechanica*) and Archimedes (*On the equilibrium of planes*). There it was seen that weight and length may compensate each other, so that less weight can be compensated by more length (Renn/McLaughlin 2018; McLaughlin/Schlaudt 2020). Since weight and length are different and thus not directly comparable dimensions, it is anything but trivial how this insight can be conceptualized as a combined “effect” of weight *and* length, although in retrospect this difficulty is obscured by the simplicity of the mathematical solution of the problem in the simple product of length and weight (in the law of the lever). A more recent example is the concept of (kinetic) energy. It was not the case that physicists first introduced a measure for a precisely identified quantity – “energy” – and then studied it quantitatively only to discover that they were dealing with quite a special quantity to which a conservation law applies. In fact, conservation of energy in mechanical systems was by no means a discovery about energy. Rather, the physicists of the 17th century agreed that something which they provisionally called the “force of moving bodies”, and which would later be termed “energy”, must be preserved in mechanical systems.¹ In the so-called *vis-viva controversy*, they then argued about how this force should be “estimated”, i.e. how its measure could be defined in such a way that its conservation would be expressed in the measure’s numerical constancy (McLaughlin 1996). Conservation was thus something of a constraint on the quantification of energy. Additional complications arose from the fact that, as we know in retrospect, two different characteristic quantities, each one obeying a conservation principle, are involved in what was referred to as the “force” of a moving body – two different quantities which first had to be separated from each other, viz. energy and momentum, the former a scalar quantity, the latter a vectorial quantity, for which the adequate mathematical means of representation were also lacking. A similar case is known from the development of the thermometer, in which “temperature” and

“quantity of heat” first had to be conceptually separated (Böhme 1976b; 1999). All these cases show that the formation of quantitative terms, even if it eventually results in a simple and precise definition, is a historically complicated process occurring in a space defined by prior knowledge, preliminary concepts of the phenomenon, and the technical and mathematical means available – a process lacking a priori identifiable conditions of success (Schlaudt 2009b).

In this context, it is also remarkable that the description of measurement technology and physical theory formation flow smoothly into each other. Antiquity, the Middle Ages and modern times all knew literature dedicated to practical measurement, such as the *Surveyor’s book* of Hyginus Gromaticus (Hyginus 2018) from the first century A.D. or Albrecht Dürer’s *Vnderweysung der Messung [Instructions in Measurement]* from 1525 or Johannes Kepler’s *Auszug auss der uralten Messe-Kunst Archimedis [Excerpt from Archimedes’ ancient art of measurement]* from 1616 and George Adam’s *Geometrical and Graphical Essays, containing a description of the mathematical instruments used in geometry, civil and military surveying, levelling and perspective* from 1791. The medieval *Art of Weighing or scientia de ponderibus* (Moody/Clagett 1952), in which Duhem locates the beginnings of statics (Duhem 1905/1906), also belongs in this context. Simon Stevin’s work from 1586 was still entitled *Principles of the Art of Weighing (De Beghinselen der Weeghconst)* yet no longer deals with metrology but with theoretical statics. This connection can still be found in modern physics. Einstein’s fundamental work on special relativity, *Zur Elektrodynamik bewegter Körper* (1905), begins with a revision of the basic concepts of time and space and their measurement. Metrological foundations and physical theory formation thus remain in a complicated interrelation.

In other sciences, which could not rely on an existing stock of prescientific measuring techniques but had to construct their basic quantitative concepts from scratch, it can be seen that the difficulties in doing so can already occur at a very basic level, sometimes even in simple counting. In the construction of measures of local biodiversity, for example (Magurran 2004: 72,

¹ According to Freudenthal 1999, the fact that energy must be preserved can be understood as a consequence

of the concept of a physical system, which simply includes energy conservation as a criterion of its identity.

137 f., 142 f.), it must be clarified in advance what is to be understood by a species at all, and even then problems remain, for example because not all species are permanently resident in one place. In plant collectives that have been created by vegetative reproduction and thus consist of clones (so-called genes or *clonal colonies*), even the identification of an individual of the species is problematic. Likewise, it is not always clear to what extent independent entities can be identified in ecosystems in which the elements support each other as a result of an evolutionary stabilization, as is the case today in approaches such as ‘green accounting’ (Vatn 2000). In the development of measures of economic capacity such as gross domestic product, it must be clarified which is the smallest productive unit. Econometrics opted for the household rather than the individual (Gilbert et al. 1949), which means that only goods and services exchanged via the market are counted whereas domestic production falls out of account (Waring 1988). These examples show that even before coming to proper metrological problems, preliminary questions of comparability and the unambiguous identification of units necessary for mere counting can be problematic.

3. Implications for Natural Philosophy

Regarding the meaning, and the consequences, of quantification and measurement from the point of view of natural philosophy, two perspectives can provisionally be distinguished, one of which focusses on the *result of the measurement*, i.e. the mathematical representation, and the other on measurement as a *process*.

Starting from the mathematical representation that resulted from measurement – and fueled by the considerable success of a mathematical description of

nature in physics – some authors put forward the idea of a mathematical constitution of nature itself. The physicist Eugene P. Wigner concluded from the success of the mathematical description of nature that mathematics is “in a very real sense, the correct language” (Wigner 1960: 8). While “correctness” probably means the structural similarity of mathematics and nature, Werner Heisenberg went a step further and identified the building blocks of matter with mathematical entities. The fact that the properties of elementary particles can be obtained as invariants of group-theoretically described symmetry transformations of a field theory suggested to him that the elementary particles should simply be identified with these symmetries: “For the smallest units of matter are in fact not physical objects in the ordinary sense of the word; they are forms, structures, or in the sense of Plato, ideas about which one can speak unambiguously only in the language of mathematics” (Heisenberg [1967] 1973: 237). Such a form of Platonism and mathematical realism can be traced back to the Renaissance, for example to Galileo’s famous comparison of the universe with a book written in the “language of mathematics”, with “triangles, circles, and geometric forms as its characters” (Galileo [1623] 1977: 25;² for the historical context see Gorham et al. 2016; Falkenburg 2017).

In the second perspective, which is based on measurement as a process, the instrument’s mediation of the experience of nature comes to the fore. Here, three aspects are particularly relevant: the relationship between nature and technology, quantity as a dispositional property, and the unit of measurement as a natural object.

The first aspect concerns a point which measurement has in common with experiment. Measurements are done with measuring devices and thus are technically

² Galilei [1623] 1977: 25: “La Filosofia è scritta in questo grandissimo libro, che continuamente ci stà aperto innanzi à gli occhi (io dico l’virsuo) ma non si può intendere se prima non s’ipara ‘a’itender la lingua, e conosceri i caratteri, ne’quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, & altre figure Geometriche, senza i quali mezi è impossibile à intenderne vmanamente parola; senza questi è vn’aggirarsi vanamente per vn’ocuro laberinto.” English translation in Galileo 1957: 237 f.:

“Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single workd of it; without these, one wanders about in a dark labyrinth.”

mediated in the same way as experiments. As for experiment in general, measurement in particular can only be considered as a means for acquiring scientific knowledge insofar as the (generally implicit) concept of nature assumes that technology is part of nature. In Aristotle's *Mechanical Problems* (Aristotle: *Mechanica*; Winter 2007), for example, there is a marked opposition between nature (*φύσις*, *physis*) and technology (*τέχνη*, *techne*) (cf. Dunshirn 2019). Nature comprises processes that happen by themselves ("things that happen according to nature") and which, accordingly, can only be recorded through observation. Technology, on the other hand, is regarded as unnatural or even as a 'trick' on nature ("things happening contrary to nature, done through art for the advantage of humanity"). In technology, humans accomplish what would not have happened by itself, and under certain circumstances humans produce effects that actually exceed their natural powers. In early modern times the concept of nature has changed. With Francis Bacon, "natural" and "artificial" cease to form a contrast. In the descriptions of the technologies of *New Atlantis* (Bacon 1626), "natural/naturalis" and "artificial/artificialis" now only refer respectively to the natural or human origin of, for example, a metal or a mineral, but no longer to an ontological difference. René Descartes explicitly identifies art and nature with each other in the *Principles of Philosophy* of 1644: apart from the size of the parts, he acknowledges no difference between machines produced by craftsmen and bodies produced by nature; all the rules of mechanics (i.e. the science of machines) thus also belong to physics (the science of nature) so that, as the contemporary French translation states even more clearly, "all things that are artificial are therefore also natural".³ This subsumption of technology under nature is necessary for understanding the experiment in such a way that nature is revealed in technology. Insofar as experiment and measurement become the privileged access to nature in the sciences, the subsumption of technology under nature can reverse itself into that of

nature under technology. Nature is then interpreted "*sub specie machinae*" (Freudenthal 1999: 17), i.e. as a machine – usually a clock – for example in the idea of a *machina mundi* or world machine (McLaughlin 1994).

The second aspect relevant to natural philosophy (and directly related to the first) is that of quantity as a dispositional property. Measurement involves a causal interaction between the measured object (the "*measurand*") and the measuring instrument. The instrument plays a twofold role. First, it acts as a filter that reduces the possible causal interactions to one dimension, namely the dimension of human interest in the measurement. For example, on a set of weighing scales, a body acts only in terms of its weight, in the interaction with an electrometer only in terms of its electric charge, etc. Secondly, the instrument has a mechanism that allows us to determine the interaction numerically. Measurement as a causal interaction means, however, that the quantity determined in the measurement has the character of a potency or even a disposition (on fundamental physical properties as dispositions see Goodman 1983: 40 f., 45). The numerical value has therefore to be understood as a prediction of the effect an object will produce on a corresponding, standardized measuring instrument. Measurement presupposes control, and thus tells us nothing about "untouched" nature, but about nature insofar as it is under our control.

The quantities of length and duration occupy a special position in this respect, since they are not measured merely as determinations of objects and events but are hypostasized, independently of material points of reference, to the dimensions of space and time as such (or a four-dimensional space-time continuum, cf. Minkowski 1909). Newton described space and time, as they underlie physics, as entities characterized by an inner homogeneity. It is clear that this is a projection of technical standards of measurement onto nature. A *clock* must run evenly and *yardsticks* must be transported without deformation and placed together without gaps. This results in a time that flows evenly and a

³ Descartes 1644: §§203, 307: "Atque ad hoc arte facta non parum me adjuverunt: nullum enim aliud, inter ipsa & corpora naturalia discrimen agnosco, nisi quod arte factorum operationis, ut plurimum peraguntur instrumentis adeò magnis, ut sensu facilè percipi possint. Et sanè nullæ sunt in

Mechanicâ rationes, quæ non etiam ad Physicam, cujus pars vel species est, pertineant". Cf. Descartes 1647: 480: "Et il est certain que toutes les regles des Mechaniques appartiennent à la Physique, en sorte que toutes les choses qui sont artificielles sont avec cela naturelles."

space that is continuous and homogeneous (cf. Janich 1985). James J. Gibson can thus legitimately assert from the standpoint of empirical psychology of perception: “Isaac Newton asserted that ‘absolute, true, mathematical time, of itself and from its own nature, flows equably without relation to anything external.’ But this is a convenient myth. Time and space are not empty receptacles to be filled; instead, they are simply the ghosts of events and surfaces” (Gibson 1979: 100 f.).

A third aspect relevant to natural philosophy can be identified in the role of the unit of measurement. There is a tension between the definition of measurement as the determination of *a* quantity (in the definition given by Stevens quoted at the beginning or in the *Vocabulaire International de Métrologie*, where measurement is defined as the “process of experimentally obtaining one or more quantity values that can reasonably be attributed to *a* quantity” – JCGM 2012: 16, my emphasis) and the practical procedure of measurement in which the relation between *two* quantities, namely the quantity to be measured and the unit, is determined. Strictly speaking, a measurement could only be regarded as the determination of *a* quantity if the quantity of the unit could be taken as known, which leads into an infinite regress, since the latter, too, would have to be determined by measurement. Most authors have simply noted but not analysed the relational nature of quantity.⁴ (Accordingly, measurement also played a key role in the transition from the substantial to the relational concept of nature in Cassirer’s *Substance and Function*, see Cassirer 1910.) Karl Marx, whose analysis of the exchange of goods treated it as a process of economic measurement, used the measuring of weights as an analogy, thereby giving a fascinating hint for further analysis of the role played by the unit (Marx [1867] 1962: 71): in measurement, the weight “counts” as mere heaviness or mere manifestation of heaviness. The weight to be measured is thus “expressed” not in the abstract quantity of the unit but in the unit itself, i.e. in a material body. Conversely, the unit is not merely a natural body

regarded as an instantiation of an abstract quantity but rather a body that is reduced to this property. In the concrete unit the abstract quantity can be touched and manipulated. Here, quantification again becomes visible as the challenge of establishing a consistency between the algebraic handling of symbolic quantities and the technical handling of concrete quantities – in short, the challenge of developing a technique for handling natural objects according to the rules of mathematics. Engster (2020: 265) therefore interprets measurement not only as the construction of knowledge about natural objects but as construction of these natural objects themselves. Measurement is understood by him as a twofold action: first, to “extract” and to “exclude” the units of measure from nature, then to “[hold nature] to its own measures and [force it] to determinate itself”. Nature thus “becomes an object measured by a specific part of itself”, and measurements can be characterized as “nature’s self-reflection”.

4. Measurement and Science

The importance of measurement (and hence of mathematics) for science is still an open question. Kant explained it apodictically: “I assert, however, that in any special doctrine of nature there can be only as much *proper* science as there is *mathematics* therein” (Kant [1786] 2002: 185). In the post-Kantian, ‘Romantic’ philosophy of nature this question became controversial. In his work *Kosmos*, Alexander von Humboldt explains: “Man cannot act on nature, or appropriate her forces to his own use, without comprehending their full extent, and having an intimate acquaintance with the laws of the physical world” (Humboldt 1849: 34⁵). In a letter he virtually confesses to the “fury of numbers”: “J’ai la fureur des chiffres exacts” (quoted in Knobloch 2006: 62). Goethe, on the other hand, insisted in his *Theory of Colours* that “physics exists independently of mathematics” (Goethe 1833: 304) and polemicized e.g. against Tycho Brahe as a representative of quantitative physics:

⁴ Edgar Wind (1934: §1) writing on the “circle of research” is not concerned with the problem of how a quantity becomes epistemically accessible through its relationship to another quantity (the unit), but how such a comparison can produce “exact” knowledge.

⁵ The original German text emphasizes the *quantitative* form of this scientific knowledge, cf. Humboldt 1845: 36.

“Thus he proceeds on the occasion of colour, which he treats only in passing, because to him, for whom everything is measure and number, it can be of no importance” (Goethe 1810: Vol. 2, 283; cf. Böhme 1977: 34; Müller 2015). According to Ruben (1975: 35 f.) it is characteristic of the Romantic philosophy of nature to “turn against the analytical style and its affinity to calculation, against its reduction of sensual experience of nature to analytical calculation.” The criticism of science and technology by the Frankfurt School follows in this tradition. Georg Lukács, for example, understands the “calculability” or “predictability” inscribed in the modern rationalization of economic production as “a rupture with the organic-irrational, always qualitatively determined unity of the product itself” (Lukács 1923: 99). Erich Fromm, too, critically assesses the abstraction underlying quantification and measurement by emphasizing – undoubtedly with Marx’s distinction between “value” (in the sense of economic value, exchange value) and “use-value” in mind – its intuitive contrast with the qualitative, living, emotional-intuitive, unpredictable and authentic (Fromm [1956] 1991: 107–111). In this light, quantification does not appear merely as fetishism and reification (in the Marxian sense) but is interpreted by Fromm socio-psychologically as an expression of “necrophilia,” i.e., an attraction to the lifeless (Funk 2011).

However, the question of the role of measurement and mathematics in science can be clearly answered neither systematically nor historically. It can only be noted that some disciplines in the social sciences, cultural sciences and life sciences – above all psychology and economics – have accepted mathematization as an ideal, though this orientation is not undisputed within the disciplines (for economics, see Mirowski 1989). Even when in critical debates about quantification the notion of an originally qualitative phenomena that inherently refuses to be quantified is often cited, this argumentation remains problematic simply because quality and quantity do not form a contradiction: measurement is rather the quantitative expression of a qualitatively determined dimension (such as gravity, length, temperature, etc.). In the current state of knowledge, measurement can neither be asserted as necessary for the empirical sciences, nor, conversely, can an a priori limits to quantification be defended; instead, only the success and failure of specific

attempts at quantification can be registered on a pragmatic level (Michell 1997; Berka 1983: 208; Schlaudt 2009b: 225–229).

Attempts to quantify economic performance are currently the subject of particularly controversial discussion, as they are becoming relevant as indicators in practical contexts. Two examples are particularly relevant. One is quality control, which was first introduced in private companies and has been transferred to public institutions in countries such as the USA, Great Britain and France within the framework of so-called *New Public Management*. The other is indicators that are supposed to show the prosperity or the degree of democratization and the like of nations and economies. In recent years, an extensive literature has emerged on these fields, analysing them in historical and sociological terms, often taking a critical stance (Merry 2016; Supiot 2017; Muller 2018; Schlaudt 2018).

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