School Tablet with Divisions and Demonstrations in the Chester Beatty Library

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§1 The wax tablet published here (CBL 142.2) is inscribed with two series of divisions for the divisors 6 and 7. While division tables, also called tables of fractions or tables of parts, comprise the most common kind of arithmetical tables preserved in papyrological evidence, the Chester Beatty tablet belongs to a peculiar subtype that includes demonstrations. In these tables, entries for divisions follow the conventional format in which a series for each divisor n begins with the division of 6000 and then lists divisions and their quotients for whole-number dividends from 1 to 10, then for tens, then hundreds and then thousands up to 10,000 (one myriad) for $n \leq 10$ and up to n for $n > 10$. Unlike in most division tables, however, every quotient is accompanied by a series of multiplications in which each constituent part of the quotient is multiplied by the divisor with the product of each multiplication recorded. Since the sum of products in each series of multiplications amounts to the dividend, the procedure demonstrates that the quotient is correct. Consequently, these reverse computations are sometimes called verifications.[1]

§2 Since division entries accompanied by demonstrations tend to require more space on a line, tablets with such divisions, including CBL 142.2, have a horizontal orientation, with writing parallel to the longer side of the tablet. The horizontal layout is preferred even when other leaves in the same codex have a vertical orientation.[2] Although arithmetical tables in general are difficult to date with precision, it appears that tablets inscribed with division-with-demonstration tables tend not to predate the fifth century. I am currently aware of the following examples (dimensions are given width × height):

- Louvre AF 1196[2] Face B + Louvre AF 1196[3] Face B (⧉ SB 20 14649), 5th–6th c., division table for 2/3. Inked tablets, 26 × 12.5 cm and 26.5 × 12.5 cm respectively.[3]
- ⧉ T.Varie 78, 5th c. (?), division table for 17. Wax tablet, c. 20 × 10 cm.
- ⧉ T.Varie 16 = P.Vat.Gr. 60, 6th c., division tables for 15, 16, 17, 18. Wax tablet, 23.5 × 13.6 cm.
- ⧉ T.Varie 17 = P.Vat.Gr. 61A, 6th c., division table for 19. Wax tablet, 23 ×12.5 cm.
- ⧉ T.Varie 4–5 = P.Vat.Gr. 53 A–B, 7th c., division tables for 2 and 4. Inked tablet, 47 × 22.7 cm.
- ⧉ T.Varie 7 = P.Vat.Gr. 55A, 7th c., division tables for 17 and 19. Inked tablet, 39.3 × 15.7 cm.
- ⧉ Würzburg, Martin-von-Wagner Museum K 1024 (TM 69464), 8th c., division tables for 2/3 and 11. Inked tablet, 40.7 × 18.7 cm.[4]

§3 In some of these tables, the demonstration part is visually separated from the quotient. Thus, in T.Varie 78, most entries have a space about a letter or two wide after the quotient. In T.Varie 4 and 5, each division has a tripartite entry arranged in three columns: the dividend, the quotient, and the demonstration. Whether such division-with-demonstration tables served a didactic purpose (for example, as a teacher's model) or were results of exercises is impossible to deduce from the content alone, but the medium and style of writing might offer some clues. Large neatly inscribed tablets like T.Varie 4–5 might well have been models intended to help expound the calculations. Wax tablets, on the other hand, were likely used for students' exercises in which computational skills were trained

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1 See ⧉ Cauderlier 1983: 267–268; Brashear (⧉ 1984b: 216), on the other hand, suggests that these multiplications might reflect the method of computing fractions that had roots in pharaonic Egypt.

2 This is the case, for example, with the codex in the Morgan Library, T.Varie 71–78, in which all tablets but the one with the divisions (T.Varie 78) have vertical orientation.

3 For the ed.pr. of both tablets AF 11962 and 11963 see ⧉ Boyaval 1973: 243–256. Cauderlier (⧉ 1983: 261, 266–268) offers improvements to the ed.pr. and identifies the faint traces of writing on 11963 Face B as continuation of the division table for 2/3 inscribed on AF 11962 Face B; for AF 11962 Face B cf. also Brashear (⧉ 1984b: 215–217). Online images of AF 11962 can be found at ⧉ https://collections.louvre.fr/en/ark:/53355/cl010048857 and of AF 11963 at ⧉ https://collections.louvre.fr/en/ark:/53355/cl010002616.

4 For the edition of this tablet, see ⧉ Brashear 1984a.

either by copying, which was possibly meant to help understand the division, or by performing actual calculations. As far as the tablet published here is concerned, the type of medium (i.e. wax tablet), the gap in the series of divisions (see below), and the untidy character of inscribing suggest that it came from a school environment and was a product of an exercise.

§4 According to the 'Recent acquisitions' notebook of the Chester Beaty Library, the tablet was purchased in Cairo in July of 1938, but no further details of its provenance are known. Both sides of the tablet have a recess panel that was covered with wax to create the writing surface, which indicates that it was an inner leaf of a wooden codex. It was tied to other leaves by a string threaded through the four holes drilled on one of the long sides, which is the upper side in relation to the writing. On the spine edge of the tablet four notches can be seen.[5] It is unclear whether their purpose was to keep the string tying the leaves in place or to serve as collational marks to indicate the position of the tablet in relation to other leaves.[6] The former seems somewhat likelier not only because the placement of the notches corresponds to the holes, but also because they are cut straight and not at an angle. Another, slanting notch is cut into the fore edge of the table. That notch surely was a collational mark.[7]

§5 A single hand inscribed both sides of the tablet, and, as was usual for writing on wax, it did not employ ligatures.[8] Letters at the beginnings of lines on either side are drawn more carefully and uniformly than towards the ends of the lines, where they vary more in size and shape. Where writing has spilled into the margins not covered by wax, letters are scratched directly into the wood and become angular and very uneven. Paleographically, the more neatly inscribed parts find close parallels in T.Varie 16 and 17, which also contain division tables and display similar small roundish letters and the same symbol for 2/3, which looks like omega with a long vertical at its right-hand side and resembles Coptic *shai*. The date in the sixth century assigned to T.Varie 16 and 17 can be reasonably conjectured for the tablet published here, too.

§6 Side A contains a table of divisions by 6 inscribed in two uneven columns separated by a line drawn somewhat left of center. After the header featuring the usual division of 6000, the first column has entries for dividends from 1 to 60, while the second begins with 200, and the divisions up to the dividend 9000 can be discerned. The reason for the gap in the series—with entries for 70, 80, 90 and 100 apparently missing—is not clear. While it is theoretically conceivable that one or even two entries may have been squeezed at the lower or upper edge of the tablet and eventually lost, it is highly unlikely that all four could have fit, and I am inclined to think that they were never inscribed. Side B comprises divisions by 7 for consecutive dividends from 1 to 50.

§7 For each series, the divisor is stated explicitly only in the heading. All subsequent entries contain the dividend in the form τῶν N, 'of N', where τὸ ϛ and τὸ ζ τῶν N, '1/6 and 1/7 of N', are implied for divisions by 6 and 7, respectively. The dividend is followed by the quotient and then by a demonstration. In the demonstration, the quotient is decomposed into its denary components and fractional parts, with each component multiplied by the divisor. For example, the division of 200 by 6 is presented in the following way:

τῶν σ λγγ′ λ ϛ ρπ· γ ϛ ιη· γ′ β of 200 is 33 1/3 30×6=180, 3×6=18, 1/3(×6=) 2

where τῶν σ λγγ′ is the division '(1/6) of 200 (is) 33 1/3', and λ ϛ ρπ· γ ϛ ιη· γ′ β, '30×6=180, 3×6=18, 1/3(×6=) 2' is the demonstration, in which each element of the decomposed quotient (i.e. 30 + 3 + 1/3) is multiplied by the divisor. Since the sum of the products amounts to the dividend (180 + 18 +

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5   For these notches, see the photo at ⧉ https://cbl01.intranda.com/viewer/image/W_142_2/3/.

6   For a discussion of collational marks in wooden codices, see ⧉ Sharpe 1992, esp. 132–133.

7   For the photo of that edge, see ⧉ https://cbl01.intranda.com/viewer/image/W_142_2/5/. For similarly looking single notches on the fore edge of wooden codex P.Berol. 14000, cf. ⧉ Plaumann 1913: 210–219 with Abb. 96; the images of the codex are available at ⧉ https://berlpap.smb.museum/05451, but they do not show the collational marks visible in Abb. 96 of the edition.

8   Cf. introduction to T.Varie 71–78, p. 156.

2 = 200), the computation can be viewed as demonstrating the correctness of the division. The sum, however, is never recorded.

§8 It is noteworthy that in the demonstration part the multiplier (i.e. the divisor 6 or 7) is recorded explicitly for the multiplicands (i.e. the elements of the decomposed quotient) that are whole numbers above 1, but not for fractions or 1. Thus, in the example above, multiplier 6 is recorded with the multiplicands 30 and 3, but not with 1/3. Perhaps the difference was owed to considering what we see as '1/3 times 6' not as a multiplication, but as a part, i.e. 1/3 of 6, where the whole number was viewed for some reason as not needing a repetition.

§9 The tablet's wax surface is not well preserved. Random scratches and loss of wax make it in places hard to discern specific letter forms or even to see any writing at all. For example, I cannot ascertain how, or even whether, the digits for thousands are marked (Side A col. ii, lines 9–15). It might well be that they were not indicated at all. Nor can I determine if all the instances of the fraction 1/7 on Side B are distinguished from the number 7, although symbols for 1/3 (γ′) and 1/6 (ϛ′) on Side A clearly show a tick-like extension in the upper right corner of the respective letters. There also seem to be overstrokes above the last digit of two-digit fractions, see e.g. κη̄ for 1/28 on Side B, line 10, but it is unclear if this was done consistently. Most regrettably, the heading to the series on Side A is badly damaged and almost completely lost on Side B, so that it is impossible to see whether they featured a demonstration for the quotients in the division of 6000 (see note to Side A col. i, line 1).

§10 On both sides, the entries for the divisions run more or less horizontally and parallel both to each other and to the edge of the tablet, but the entries for demonstrations, which form a separate column, are less regular and increasingly slant downwards. In the first column of Side A, for example, the demonstrations from about line 11 on are no longer on a plane with the quotients for which they serve as verifications. This is especially apparent at the bottom where the waxed writing surface abuts the frame. At the top of the edge of the frame, the sequence τῶν μ ϛω εϛλ 'of 40 is 6 2/3 5×6=30' can be read; it corresponds to the entry for the division of 40 (l. 14), τῶν μ ϛω, 'of 40 is 6 2/3', followed by the demonstration of the division of 30 from line 13: ε ϛ λ, '5×6 = 30'.

§11 In the edition below, I separate multiplications in the demonstration part by a colon in the Greek text and by a comma in the translation in order to visualize the computation steps. In the translation I also supply in parentheses the implicit multipliers (which equal the divisors 6 and 7 in the two respective series) and, when there is more than one multiplication, the sum of the products (which amounts to the dividend in the corresponding division). Signs for arithmetic operations are put in parentheses only when they go with implicit multipliers, but otherwise are printed outside them. The ticks and overstrokes are shown only where discernible on the tablet.

CBL W 142.2	17.5 (h) × 11.9 (w)	Sixth century (?)
	Wax tablet	Provenance unknown

 Figure 1. CBL W 142 Side A. Chester Beatty CC BY – 4.0
 Figure 2. CBL W 142 Side B. Chester Beatty CC BY – 4.0

A

i

† [τ]ὸ ϛ′ ἐν ψήφων α

	τῶν α	ϛ′	ϛ′ α
	τῶν β	γ′	γ′ β
	τῶν γ	ζ	ζ γ
5	τῶν δ	ώ	ώ δ

	τῶν ε	∠γ′	∠ γ· γ′ [β]
	τῶν ϛ	ἀ	α ϛ
	τῶν ζ	αϛ′	ἀ ϛ· ϛ′ [α]
	τῶν η	αγ′	ἀ ϛ· γ′ β
10	τῶν θ	α∠	α ϛ· ∠ γ
	τῶν ι	αωϛ	α ϛ· ωϛ δ
	τῶν κ	⟦α⟧ γγ′	γ ϛ ιη· γ′ β
	τῶν λ	ε	ε ϛ λ
	τῶν μ	ϛωϛ	ϛϛ ϛ λϛ· ωϛ δ
15	τῶ[ν ν]	[ηγ′]	η ϛ μη· γ′ [β]
	τῶ̂[ν ξ]	[ι]	ι ϛ ξ

ii

	τῶν σ	λγγ	λ ϛ ρπ· γ ϛ ιη· γ β
	τῶν τ	ν	ν ϛ τ
	τῶν υ	ξϛωϛ	ξ ϛ τξ· ϛ ϛ λϛ· ωϛ δ
	τῶν φ	πγγ′	π ϛ υπ· γ ϛ γ ιη· γ′ β
5	τῶν χ	ρ	[ρ] ϛ χ
	τῶν [ψ]	[ρ]ιϛωϛ	ρ ϛ χ· [ι] ϛ ξ· ϛ ϛ λϛ·ωϛ δ
	τῶ[ν ω]	ρλγγ′	ρ ϛ χ· λ ϛ ρπ· γ ϛ ιη·γ′ β
	τῶ[ν λ͞]	ρ[ν]	[ρ] ϛ χ· ν ϛ τ
	τῶ̂ν Α	ρ[ξ]ϛωϛ	[ρ] ϛ χ· ξ ϛ τξ· ϛ ϛ λϛ·ωϛ δ
10	τῶν Β	τλγ[γ′]	τ ϛ Αω· λ ϛ ρπ· γ ϛ ιη· γ′ [β]
	τῶν Γ	φ	φ ϛ Γ
	τῶν Δ	χ[ξϛ]ωϛ	χ ϛ Γχ· ξ ϛ τξ· ϛ ϛ λϛ· ωϛ [δ]
	τῶν Ε̇	[ωλ]γγ′	ω ϛ Δωϛ· λ ϛ ρπ· γ ϛ ιη· γ′ [β]
	τῶν Ϛ̇	Α	Α ϛ Ϛ
15	τῶν Ζ̇	[Α]ρξϛωϛ	Α ϛ Ϛ· [ρ] ϛ χ· ξ ϛ τξ· ϛ ϛ λϛ· ωϛ [δ]
	τῶ̂[ν Η̇]	[Ατ]λγγ′	[Α ϛ] Ϛ· τ ϛ [Α]ω· [λ ϛ ρπ·] γ ϛ [ιη· γ′ β]
	τῶ̂[ν] Θ	Αφ	Α [ϛ Ϛ]· φ ϛ Γ

B

-ca.?- [-ca.?-] ωνζ [ζ]

	τ[ ] α	ζ	[ζ] α
	τῶν̇ β̇	ḋ κη͞	d α∠d· κη͞ [d]
	τῶν γ̇	γ̇′ [ιδ̇ μβ]	[γ′] βγ′· ιδ ∠· [μβ ϛ′]
5	τῶν δ	[∠ ιδ]	∠ γ∠· [ιδ ∠]
	τῶν ε̇	ωϛ κα	[ωϛ δωϛ· κα γ′]
	τῶν ϛ	∠ γ′ μβ	∠ γ[∠· γ′ βγ′· μβ ϛ′]
	τῶν ζ	ἀ	ἀ ζ
	τῶν η	α ζ′	α ζ· ζ′ [α]
10	τῶν θ	α [d] κη͞	α ζ· d α∠d· κη͞ d
	τῶν ι	α γ′ ιδ μβ͞	α ζ· γ′ βγ′· ιδ∠· μ[β] [ϛ′]
	τῶν κ	β ∠ γ′ μβ͞	β ζ ιδ· ∠ γ∠· γ′ βγ′· μβ ϛ
	τῶν λ	δ ḋ κη͞	δ ζ κη· d α[∠]d· κη ḋ
	τῶν μ̇	ε̇ ωϛ κα	ε̇ ζ λε̇· ωϛ [δωϛ]· κα γ′
15	τῶ̇ν̇ ν	ζ̇ ζ̇	ζ ζ μθ· ζ [α]

A

i

1/6 of 6000 is 1000 ...

of 1 is 1/6	1/6(×6=) 1
of 2 is 1/3	1/3(×6=) 2
of 3 is 1/2	1/2(×6=) 3
5 of 4 is 2/3	2/3(×6=) 4
of 5 is 1/2 1/3	1/2(×6=) 3, 1/3(×6=) 2, (3+2=5)
of 6 is 1	1(×6=) 6
of 7 is 1 1/6	1(×6=) 6, 1/6 (×6=) 1, (6+1=7)
of 8 is 1 1/3	1(×6=) 6, 1/3(×6=) 2, (6+2=8)
10 of 9 is 1 1/2	1(×6=) 6, 1/2(×6=) 3, (6+3=9)
of 10 is 1 2/3	1(×6=) 6, 2/3(×6=) 4, (6+4=10)
of 20 is 3 1/3	3×6=18, 1/3(×6=) 2, (18+2=20)
of 30 is 5	5×6=30
of 40 is 6 2/3	6×6=36, 2/3(×6=) 4, (36+4=40)
15 of 50 is 8 1/3	8×6=48, 1/3(×6=) 2, (48+2=50)
of 60 is 10	10×6=60

ii

of 200 is 33 1/3	30×6=180, 3×6=18, 1/3(×6=) 2, (180+18+2=200)
of 300 is 50	50×6=300
of 400 is 66 2/3	60×6=360, 6×6=36, 2/3(×6=) 4, (360+36+4=400)
of 500 is 83 1/3	80×6=480, 3×6=18, 1/3(×6=) 2, (480+18+2=500)
5 of 600 is 100	100×6=600
of 700 is 116 2/3	100×6=600, 10×6=60, 6×6=36, 2/3(×6=) 4, (600+60+36+4=700)
of 800 is 133 1/3	100×6=600, 30×6=180, 3×6=18, 1/3(×6=) 2, (600+180+18+2=800)
of 900 is 150	100×6=600, 50×6=300, (600+300=900)
1000 is 166 2/3	100×6=600, 60×6=360, 6×6=36, 2/3(×6=) 4, (600+360+36+4=1000)
10 of 2000 is 333 12/3	300×6=1800, 30×6=180, 3×6=18, 1/3(×6=) 2, (1800+180+18+2=2000)
of 3000 is 500	500×6=3000
of 4000 is 666 2/3	600×6=3600, 60×6=360, 6×6=36, 2/3(×6=) 4, (3600+360+36+4=4000)
of 5000 is 833 1/3	800×6=4800, 30×6=180, 3×6=18, 1/3(×6=) 2, (4800+180+18+2=5000)
of 6000 is 1000	1000×6=6000
15 of 7000 is 1166 2/3	1000×6=6000, 100×6=600, 60×6=360, 6×6=36, 2/3(×6=) 4, (6000+600+360+36+4=7000)
of 8000 is 1333 1/3	1000×6=6000, 300×6=1800, 30×6=180, 3×6=18, 1/3(×6=) 2, (6000+1800+180+18+2=8000)
of 9000 is 1500	1000×6=6000, 500×6=3000, (6000+3000=9000)

B

1/7 of 6000 is 857 1/7 …	
of 1 is 1/7	1/7(×7=) 1
of 2 is 1/4 1/28	1/4(×7=) 1 1/2 1/4, 1/28(×7=) 1/4, (1 1/2 1/4+1/4=2)
of 3 is 1/3 1/14 1/42	1/3(×7=) 2 1/3, 1/14(×7=) 1/2, 1/42(×7=) 1/6, (2 1/3+1/2+1/6=3)
5 of 4 is 1/2 1/14	1/2(×7=) 3 1/2, 1/14(×7=) 1/2, (3 1/2+1/2=4)
of 5 is 2/3 1/21	2/3(×7=) 4 2/3, 1/21(×7=) 1/3, (4 2/3+1/3=5)
of 6 is 1/2 1/3 1/42	1/2(×7=) 3 1/2, 1/3(×7=) 2 1/3, 1/42(×7=) 1/6, (3 1/2+2 1/3+1/6=6)
of 7 is 1	1(×7=) 7
of 8 is 7 1/7	1(×7=)7, 1/7(×7=)1, (7+1=8)
10 of 9 is 1 1/4 1/28	1(×7=) 7, 1/4(×7=) 1 1/2 1/4, 1/28(×7=) 1/4, (7+1 1/2 1/4+1/4=9)
of 10 is 1 1/3 1/14 1/42	1(×7=) 7, 1/3(×7=) 2 1/3, 1/14(×7=) 1/2, 1/42(×7=) 1/6, (7+2 1/3+1/2+1/6=10)
of 20 is 2 1/2 1/3 1/42	2×7=14, 1/2(×7=) 3 1/2, 1/3(×7=) 2 1/3, 1/42(×7=) 1/6, (14+3 1/2+2 1/3+1/6=20)
of 30 is 4 1/4 1/28	4×7=28, 1/4(×7=) 1 1/2 1/4, 1/28(×7=) 1/4, (28+1 1/2 1/4+1/4=30)
of 40 is 5 2/3 1/21	5×7=35, 2/3(×7=) 4 2/3, 1/21(×7=) 1/3, (35+4 2/3+1/3=40)
15 of 50 is 7 1/7	7(×7=) 49, 1/7(×7=) 1, (49+1=50)

Side A

Col. i

§12 **1** The expression ἐν ψήφων refers to the division of 6000 that conventionally serves as a title in late antique division tables.[9] The first line runs across the two columns, with α inscribed to the right of the dividing line. It is not clear, however, what, if anything, was written beyond ψήφων α. There is a trace of a letter further down the line and above the sigma in line 1 of col. ii (rendered by a dot in the edition) and there might be the tail of an alpha squeezed between line 1 and 2 of col. i, just above ϛ′ α in line 2 (not printed). If anything was written there, it was likely the demonstration that the quotient 1000 is correct, cf. divisions of 6000 in T.Varie 4–5, which feature such verifications.

§13 **11** Starting in this line the demonstrations start slanting down in relation to their corresponding divisions.

§14 **14–16** The demonstration ϛ ϛ λϛ· [ϖ] δ̣ in line 14 and the following two lines are scratched into the wood of the frame.

§15 **16** The division of 60 by 6 seems to be written at the very bottom of the tablet's border suggesting that entries for the dividends 70, 80, 90 and 100 were not recorded.

Col. ii

§16 **14** Α ϛ S. The demonstration is squeezed above the last digit of the quotient of the next division, [Α]ρ̣ξϛϖ.

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9 For the format of the headings in division tables, see ☑ Azzarello 2018.

§17   15 The beginning of the demonstration, Α ϛ Ṣ, written at the edge of the recess and still on the wax, is relatively well discernible, but traces after that are hard to identify conclusively until the chi, which is inscribed on the frame. Of the remaning numerals belonging to the demonstration only τξ is clear.

Side B

§18   14 The division τῶγ μ ε ο͞ κα is written at the very edge of the waxed recess, where it is followed by the first number (δ) of the demonstration for the division in line 13, which has slid down because of the continuous downward slant of the demonstration entries. The rest of that demonstration (ζ κη· ḍ α̣[ˎ]ḍ κη d) is inscribed on the wood of the frame.

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Figure 1. CBL W 142 Side A. Chester Beatty CC BY – 4.0

Figure 2. CBL W 142 Side B. Chester Beatty CC BY – 4.0

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