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On Cubic and Other Volumetric Cubits and Fingers

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§1 The recently published codex P.Math. contains a variety of mathematical problems and metrological texts.[1] These texts furnish a trove of information on metrological relations, including evidence for two peculiar volumetric units.[2] The units differ from each other and from the cubic cubit, amounting in volume to one-third or one-half of it, but irrespective of their volume each comprises 24 fingers. On the basis of the evidence from a metrological treatise of Didymus and P.Math., this paper identifies other papyrological attestations of these cubits and associates their use with mensuration of wood and timber.

1. A “regular” cubit and its subunits

§2 Many Greek papyri and manuscripts operate with a unit of length called the “cubit,” πῆχυς, which usually has no further specification, but occasionally is designated a δημόσιος, “public”, τεκτονικός, “builder’s”, or λιθικός, “stone” cubit. Its most common subdivisions are palms and fingers, and it is one-and-a-half times larger than the foot:

- 1 cubit = 6 palms = 24 fingers
- 1 foot = 4 palms = 16 fingers
- 1 cubit = 1 1/2 feet

§3 To express surface area and volume, the ancients normally used units and subunits of the appropriate order of magnitude, just as we may use 1 m (= 100 cm) for linear, 1 m^2 (= 10,000 cm^2) for surface area, and 1 m^3 (= 1,000,000 cm^3) for volume measurements. Thus, linear, square, and cubic cubits contain the corresponding power of their subunits:

- 1 (linear, εὐθυμετρικός) cubit = 24 (εὐθυμετρικοί) fingers
- 1 (square, ἐμβαδομετρικός or ἐπίπεδος) cubit = (24 × 24 =) 576 (ἐμβαδομετρικοί or ἐπίπεδοι) fingers
- 1 (cubic, στερεός) cubit = (24 × 24 × 24 =) 13,824 (στερεοί) fingers

2. The 2-palm solid cubit of Paul Tannery

§4 The metrological treatise *Mensurae marmorum ac lignorum* (On Measurement of Marble and Wood), attributed to Didymus of Alexandria, contains several mensurational problems, which feature a conversion into volumetric cubits and fingers that cannot be explained by the expected relations between

1 Bagnall and Jones 2019

2 Cf. the discussion of metrological relations in the mathematical problems in the codex, where the editors note that “[f]our problems, a3, b5, c1, and g4, present a metrological enigma,” and offer a summary of these problems and a description of the two peculiar volumetric units used in them, P.Math. p. 53-54.

the units.[3] These problems ask to find the volume of wooden geometric solids, two dimensions of which are given in fingers and one in cubits. The volume is first computed as a product of these dimensions and then is divided by 192, with the units of the resulting quotient called “solid cubits.” The remainder is then divided by 8, with the units of the quotient identified as “fingers.” The following is an illustrative example (Did. 4, Hultsch):[4]

> ξύλον στρογγύλον, οὗ τὸ μὲν μῆκος πηχῶν ιϛ´, ἡ δὲ περιφέρεια δακτύλων λ´. εὑρεῖν αὐτοῦ τὸ στερεόν· ποίει οὕτως· τὰ λ´ τῆς περιφερείας ἐφ’ ἑαυτὰ γίνονται ϡ´. ὧν τὸ ιβ´´ γίνονται οε´· ταῦτα ἐπὶ τὰ ιϛ´ γίνονται ͵ασ´· ὧν τὸ ρϙβ´´, ἵνα γένωνται πήχεις · τὰ δὲ λοιπὰ εἰς η´´, ἵνα ὦσι δάκτυλοι· ὡς εἶναι τὸ ξύλον πηχῶν στερεῶν ϛ´ δακτύλων ϛ´.

§5 “A log of wood, the length of which is 16 cubits and the circumference is 30 fingers. Find its volume. Do it as follows: (multiply) 30 of the circumference by itself, the result is 900. 1/12 of it is 75.[5] (Multiply) this by 16, the result is 1,200. (Take) 1/192 of that to convert to cubits; (divide) the remainder by 8 to convert to fingers. So that the wood is 6 solid cubits 6 fingers.”

§6 In 1881, Paul Tannery proposed an ingenious solution to explain what was going on here.[6] He determined that the unit in which the volume was expressed in this and similar problems in the treatise and which was called πῆχυς στερεός, “solid cubit,” equaled, despite its name, only one third of a cubic cubit. It was used, Tannery surmised, specifically to measure wood and corresponded to a cuboid (rectangular prism) 1 cubit × 1 cubit × 2 palms (= 8 fingers). The unit comprised 24 “(solid) fingers,” (στερεοὶ) δάκτυλοι, each corresponding to a rectangular prism 1 cubit × 1 finger × 2 palms.[7]

§7 This “solid cubit,” which I call a 2-palm solid cubit, contains 192 notional units 1 sq. finger × cubit, because 1 cubit × 24 fingers × 8 fingers = 192 sq. finger × cubit. Therefore, to convert the volume computed as the product of three linear dimensions one of which is in cubits and the other two in fingers to 2-palm solid cubits, the interim result needs to be divided by 192. The finger of the 2-palm

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3 There are three modern editions of this work, which significantly diverge from each other. The earlier one is that of Friedrich Hultsch, who included Didymus’s *Mensurae marmorum ac lignorum* in his edition of Heron, (*Heronis Alexandrini geometricorum et stereometricorum reliquiae*, Berlin 1864, p. 238-244). Johan Heiberg in *Mathematici Graeci Minores* (Copenhagen 1927) produced a different edition, in which he attempted to restore the original text of the treatise on the basis of two manuscripts that differed significantly from one another, S (cod. Constantinopolitanus Palatii veteris 1, 11th c.) and C (cod. Parisin. Gr. suppl. 387, 14th c.). Neither of these two codices was available to Hultsch, but his edition of Didymus was based on manuscripts that largely agree with C. Finally, Evert Bruins published a complete transcription of S, along with images of the manuscript and a translation, but with no apparatus criticus, in *Codex Constantinopolitanus Palatii Veteris No. 1*, Parts I-III, Leiden 1964. When citing chapters of Didymus’s work, I indicate the edition by the name of the editor in parentheses.

4 The reproduced text is based on Hultsch’s edition of Didymus, which Paul Tannery used, except that Hultsch’s substitutions of πήχεις of the manuscripts by πόδες are not repeated (cf. Tannery 1881: 158). The section corresponds to Did. 40 (Heiberg), which is based on essentially the same, albeit wordier, variant preserved in S (cod. Constantinopolitanus Palatii veteris 1, 11th c.).

5 The area (A) of the cross-section, which is a circle, is calculated by application of the usual algorithm as 1/12 of the square of the circumference (c), i.e. $A = c^2 \div 12$, which is an approximation for $A = c^2 \div 4$.

6 Tannery 1881

7 “ … l’unité de volume, quoique portant le nom de πῆχυς στερεά (sic), n’aurait été pour les bois que le tiers de la coudée cube; elle aurait donc représenté un parallélépipède rectangle ayant pour base une coudée carrée et une hauteur de deux paumes. Un doigt solide de bois aurait dès lors représenté un parallélépipède rectangle ayant une coudée de longueur, un doigt d’épaisseur, et deux paumes de hauteur,” Tannery 1881: 159.

solid cubit, which is its one-twenty-fourth part, contains 8 notional units 1 sq. finger × cubit (192 ÷ 24 = 8), which is why the remainder left after dividing the interim result by 192 is divided by 8. The drawing in Fig. 1 below provides a schematic representation of the 2-palm solid cubit and its subdivisions.

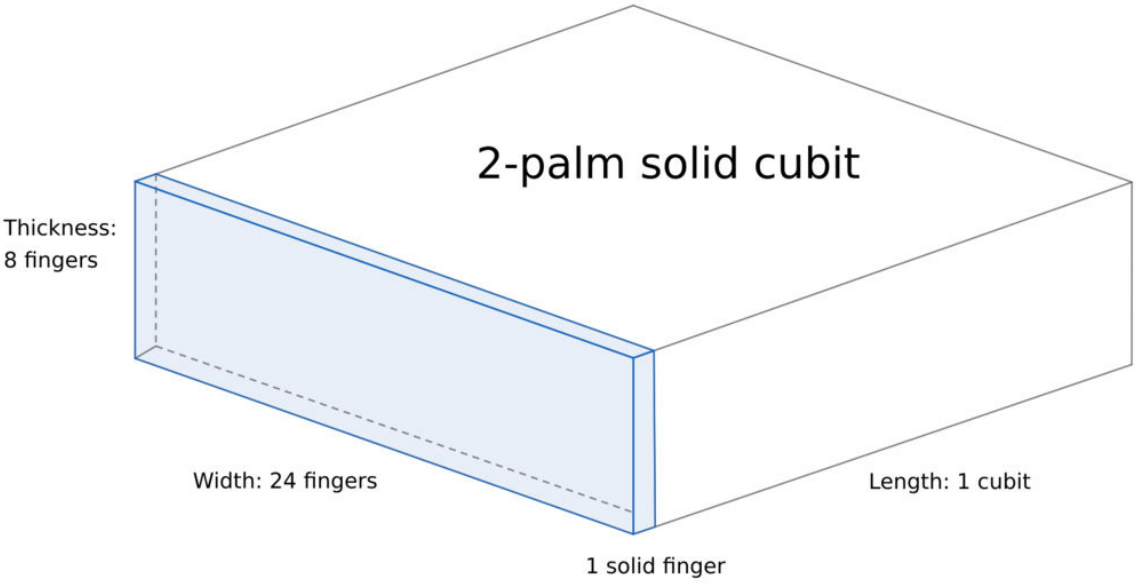


Fig. 1. 2-palm solid cubit equaling one-third of cubic cubit. Its finger is conceived of as a rectangular prism 1 cubit × 1 finger × 2 palms (= 8 fingers).

§8 Almost a century and a half after Tannery first postulated a volumetric unit called a solid cubit, which equaled only a third of a cubic cubit, the publication of P.Math. provided indisputable papyrological evidence for it. Although unnamed, it is evidently the unit used to express the volume in the solution of problem b5, P.Math. B verso:

[τρα]πέσδιον (*l.* τραπέζιον) τετράγωνον, τὸ μὲν μῆκος πηχῶν
[μ̅η̅,] τὸ τὲ (*l.* δὲ) πλάτος πηχῶν ι̅, πάχος τακτύλων (*l.* δακτύλων) ε̅,
[κο]ρηφὴ (*l.* κορυφὴ) τακτήλων (*l.* δακτύλων) β̅. οὕτω ποιοῦμεν· συν-
[τίθ]ω τὸ πλάτος καὶ τὴν κορηφύν (*l.* κορυφήν), ι̅// καὶ β̅. γί(νεται) ι̅β̅,
[ὧ̣]ν ἥμησυ (*l.* ἥμισυ), ϛ̅. πάλιν πολυπλαδιάσζωμεν (*l.* πολυπλασιάζομεν), ἐπὶ
[τ]ὰ̣ς τοῦ πάχο̣υς ε, γί(νεται) λ̅. ἐπὶ τὸ μῆκος πηχῶν
[μ̅η̅,] γ̣ί(νεται) Ἀ̅υ̅μ̅. παρὰ τὸν ρ̅ϙ̅β̅ καὶ τὰ ληπὰ ἰς τακτύλος (*l.* λοιπὰ εἰς δακτύλους).
[γί(νεται)] ζ̅ καὶ δακτύλου (*l.* δάκτυλοι) ι̅β̅. ο̣ὕτως ἔχει ὁμοίως.

§9 "A quadrangular trapezoid 48 cubits in length, 10 cubits (sic! understand "fingers") in width, 5 fingers in thickness, 2 fingers at the top. We proceed as follows. I add the width and the top, 10 and 2, totals 12, half of which is 6. Once more we multiply by 5 of the thickness, totals 30. (Multiply) by the length of 48 cubits, the result is 1,440. (Divide) by 192 and (convert) the remainder into fingers. The result is 7 (sc. solid cubits) and 12 fingers. This way for similar cases."

§10 The object is a very long trapezoidal prism, the length of which is in cubits, while the dimensions of the cross-section are in fingers.[8] To find its volume, one is instructed first to compute the surface area of the cross-section, evidently a trapeze, which is the product of the half-sum of the width and the top multiplied by the height, i.e. the thickness: $\frac{10+2}{2}$ (fingers) × 5 (fingers) = 30 (sq. fingers). This is then multiplied by the length: 30 (sq. fingers) × 48 (cubits) = 1,440 (sq. finger × cubit). This product is then divided by 192, with the remainder divided by 8, just as in the problem in Didymus, except that the units of the final result are left unnamed.

3. A 3-palm solid cubit

§11 Besides furnishing further evidence for the 2-palm solid cubit identified by Tannery, P.Math. presents a similarly conceived but hitherto unattested volumetric unit equaling one-half of a cubic cubit. The unit is deductible from computations in three mensurational problems (a3, c1, and g4), which are similar in all respects to problem b5 and the problems in Didymus except that they employ different conversion factors, that is, not 192 and 8, but 288 and 12. To account for these factors and on analogy with the 2-palm solid cubit, a 3-palm solid cubit can be postulated. It corresponds to a rectangular prism 1 cubit × 1 cubit × 3 palms (= 12 fingers) and comprises 24 (solid) fingers, each measuring 1 cubit × 1 finger × 3 palms, cf. Fig. 2.

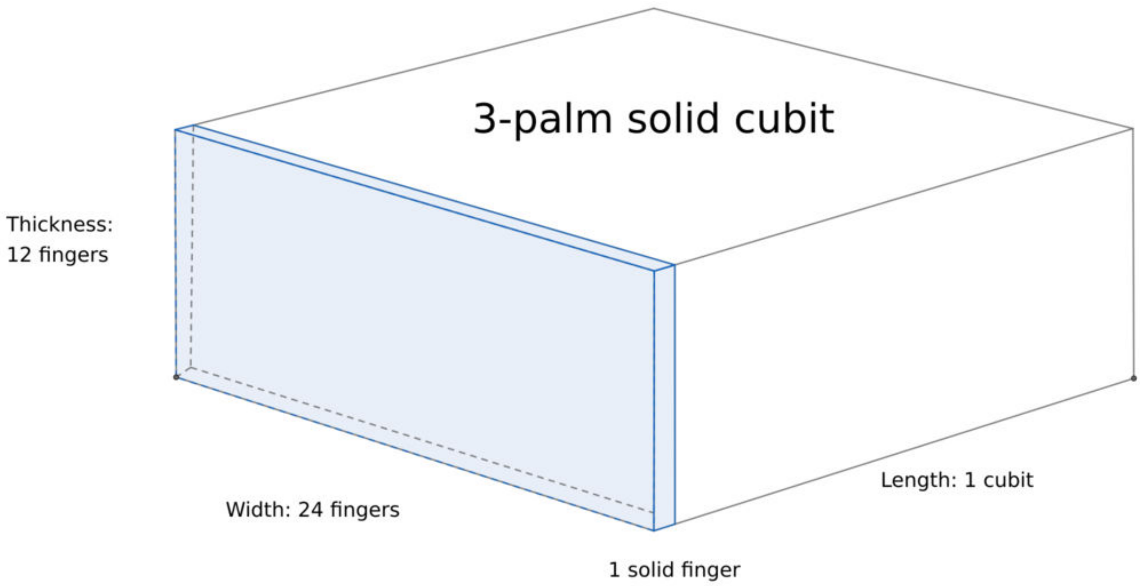


Fig. 2. 3-palm solid cubit equaling one-half of a cubic cubit. Its finger is conceived of as a rectangular prism 1 cubit × 1 finger × 3 palms (= 12 fingers).

§12 I will illustrate the conversion to the 3-palm solid cubit with problem c1 in P.Math., which fortuitously helps make sense of another papyrus, P.Harris 50, attesting the same unit. Although the statement of problem c1 is damaged, it can be deduced from its solution and the accompanying drawing. The task is to find the volume of a fresh (?) beam ξύλων (l. ξύλον) νέον (line 1), which has the shape of a quadrilateral prism with the dimensions of the cross-section given in fingers and of the length in cubits. To accomplish it, the area of the quadrilateral crossection is first computed as the product of half-sums of the opposite sides, $\frac{16+12}{2}$ (fingers) × $\frac{6+8}{2}$ (fingers) = 98 sq. fingers,[9] and then multiplied by the length (28 cubits) in order to produce the volume, 2,744 sq. finger × cubit. This result is

8 The writer mistakenly recorded width in cubits in l. 9, but the subsequent calculations make it clear that fingers are the unit meant.

9 For this algorithm, which yields only an approximate result unless the quadrilateral is a rectangle, see P.Math. p. 34.

then divided by 288, the number of sq. finger × cubit units in the 3-palm solid cubit (because 1 cubit × 24 fingers × 12 fingers = 288 sq. finger × cubit). And since the finger of this cubit, i.e. its one-twenty-fourth part, contains 288 ÷ 24 = 12 sq. finger × cubit, the remainder after the division by 288 is divided by 12 to convert it to fingers. The relevant part of problem c1 is as follows (C recto):

… οὕτω ποιοῦμαι. συντ̣ί̣[θω]
[τὸ πλ]άτος, ι̅ϛ̅ καὶ ι̅β̅, γί(νεται) κ̅η̅, ὧν ἥμισυ [ι̅δ̅. συν-]
[τί]θ̣ο̣με̣ν τὸ πάχος, {ι̅}ϛ̅ καὶ η̅. γί(νεται) ι̅δ̅, ὧν ἥμι[συ ζ̅.]
[ἐ]π̣ὶ δὲ τ[ὸ]ν ι̅δ̅, γί(νεται) ϙ̅η̅. ἐπὶ τὸ μ̣ῆκος, πηχ̣ῶν κ̅η̅,
[γί(νεται) ͵Β]ψ̅μ̣̅δ̣̅. τ̣α̣ῦτα μερίζομαι π[α]ρ̣[ὰ] τ̣ὸν σ̅π̅η̣̅,
κα̣ὶ τὰ λυπ̣ὰ̣ ἰς (*l.* λοιπὰ εἰς) δακτύλους ι̅β̅. (del. in *ed.pr.*) γί(νεται) θ̅ καὶ δακτύλ[ων]
ι̅β̅ ϖ̅. οὕτως ἔχει ὁμοίως.//

§13 “I do it the following way: I add the width, 16 and 12, totals 28, half of which is 14. We add the thickness, 6 and 8, totals 14, half of which is 7. (Multiply) by 14, totals is 98. (Multiply) by the length of 28 cubits, the result is 2,744. This I divide by 288 and divide the remainder in 12 fingers (sc. divide the remainder by 12 to convert into fingers). The result is 9 (sc. solid cubits) and 12 2/3 fingers. This way for similar cases.”

§14 Evidence for the peculiar volumetric cubits furnished by P.Math. makes it now possible to recognize the 3-palm solid cubit as the unit used to express volume in a problem in P.Harris 50 (3rd c., provenance unknown; TM 63992), and to identify 3-palm and 2-palm solid cubits with the units that are called, respectively, Ptolemaic and Nicomedian cubits in a metrological text, P.Oxy. 49 3455, lines 4-20 (3rd-4th c., TM 64339). These texts in turn provide further details about mensurational practices and the nomenclature associated with volumetric cubits of different sizes.

4. P.Harris 50 and χυδαῖοι δάκτυλοι

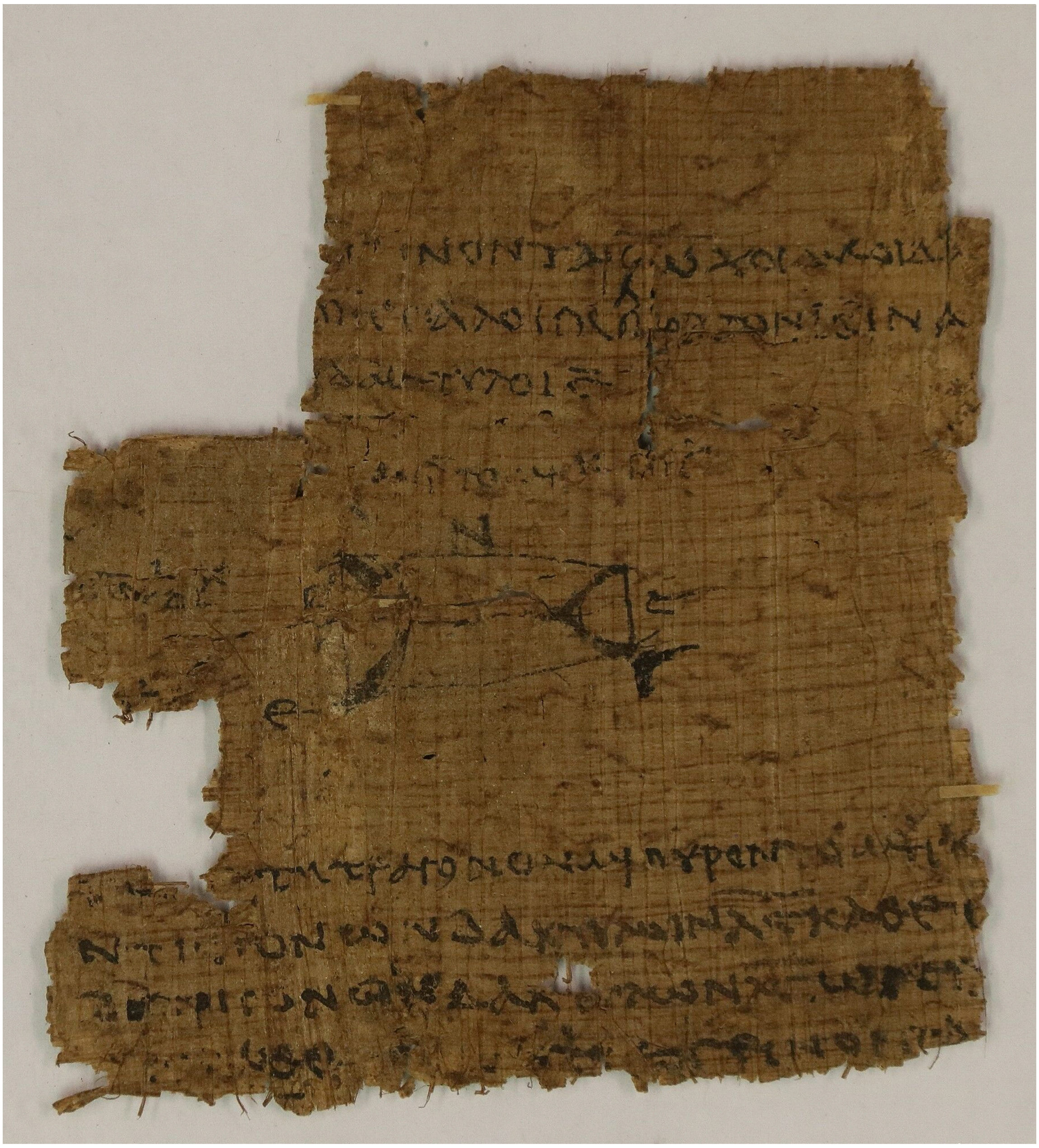


Fig. 3. Papyrus P.Harris 50. © Cadbury Research Library, University of Birmingham

§15 This small snippet of a papyrus preserves parts of two problems and a drawing. All that survives of the text of the first problem is the end of the solution, in which a result in χοιδε̂οι (l. χυδαῖοι) δάκτυλοι is divided by 288, with the remainder divided by 12 in order to convert it into a lost number of unnamed (or lost) units and fingers. The conversion factors 288 and 12 make it certain that the preserved text belonged to a volume problem and that the final result was expressed in 3-palm solid cubits and fingers.

§16 The most curious thing we learn from the fragment is the description of the units in which the interim result is recorded as χυδαῖοι δάκτυλοι. These fingers are evidently the notional units of volume computed as the product of three dimensions, two of which are in fingers and one in cubits, and which can be visualized as a prism 1 sq. finger × 1 cubit. These units are never named in the problems in

Didymus or P.Math., but they figure under the same name of χυδαῖοι δάκτυλοι in P.Oxy. 49 3455 (below). The question that presents itself is what the designation χυδαῖοι means.

§17 The answer to this question is complicated by the appearance of χυδαῖος δάκτυλος in Did. 27 (Heiberg), where it clearly refers to a surface area, not a volume, unit: ἐὰν δὲ πῆχυν ἐπὶ δάκτυλον, ποιεῖ χυδαῖον δάκτυλον α̅, ὅ ἐστι πήχεως κδ´, "when (we take) a cubit by a finger, it makes 1 χυδαῖος finger, which is 1/24 of a (square) cubit." Yet in pseudo-Heron, *Stereometrica* 1.26 (Heiberg), χυδαῖοι δάκτυλοι are again volumetric, although it is not clear what precisely they designate because the passage is hopelessly corrupt. In these two instances, Tannery translates χυδαῖος δάκτυλος as *doigt vulgaire* and Heiberg as *gemeiner* or *gewöhnlicher Zoll*, that is, "an ordinary, or common, finger." According to these interpretations, the term χυδαῖος signals that the unit "finger" is used *not stricto sensu*, but "conventionally," with another dimension or dimensions implied.[10] I wonder, however, whether it could be of significance that the word χυδαῖος is used to qualify the product of heterogenous units, fingers and cubits, computed without converting them to a single unit, but χύδην, "heaped up together indiscriminately," or "in a mixed way." If so, the adjective would indicate the non-uniform and "mixed" nature of these units, which served merely as means of computation. To express "actual" volume, the result computed in "mixed" fingers needed to be converted to solid cubits and fingers.

§18 The drawing below the volumetric problem in P.Harris 50 depicts an elongated quadrilateral with apparently parallel shorter sides, Fig. 3. Further lines within the quadrilateral signal that the object is meant as three-dimensional. Several numerals are written next to it, of which ν, ε and ϛ are quite certain; γ by the lower right corner seems to be corrected from another letter, and only traces of what is probably a β̣ are visible on the right. On analogy with problem c1 in P.Math. and assuming that β̣ is what is left of [ι]β̣ (=12), the drawing can be interpreted as presenting a quadrilateral prism with the length of 50 cubits, the width of 12 and 6 fingers, and the thickness of 5 and 3 fingers, cf. Fig. 4. In the lost solution of the problem, probably the surface area of the cross-section was first found via the usual algorithm as the product of half-sums of the opposite sides, which was then multiplied by the length to compute the volume in "mixed" fingers, $\frac{3+5}{2}$ fingers × $\frac{12+6}{2}$ fingers × 50 cubits = 1,800 "mixed" fingers (i.e. sq. finger × cubit). This result was then divided by 288 to convert it to (3-palm) solid cubits (1,800 ÷ 288 = 6 R 72), with the remainder (72) divided by 12 to convert it to fingers (72 ÷ 12 = 6).

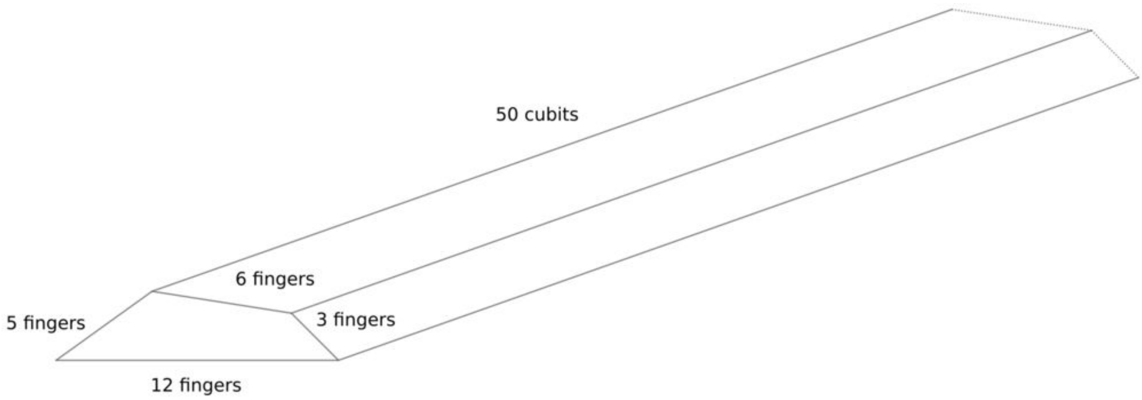
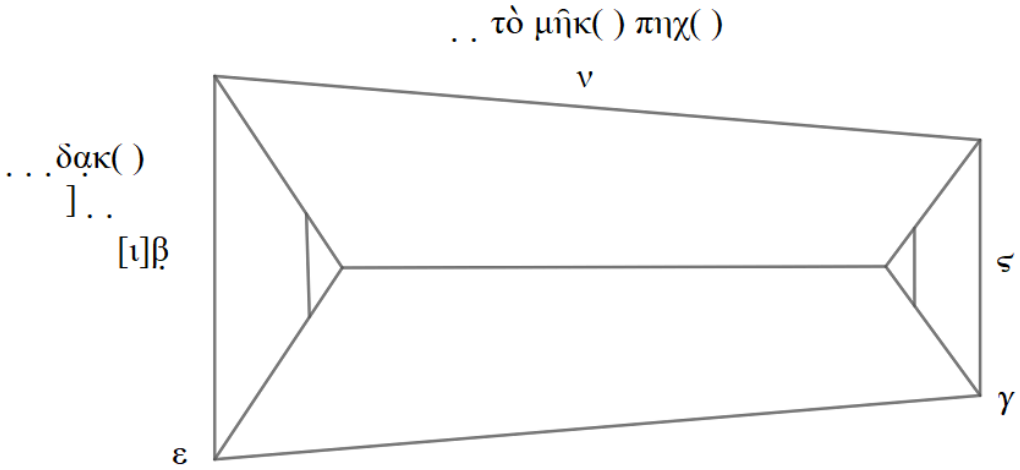


Fig. 4. Reconstructed diagram for Problem 1 in P.Harris 50.

(Problem 1) [-ca.?-] ̣ γείνονται Α̅ω̅ χοιδε͂οι δά̣κ̣[τυλοι -ca.?-]
[-ca.?- παρὰ τὸν] σπη, τὰ λοιπὰ παρὰ τὸν ι̅β̅ ἵνα [-ca.?-]
[-ca.?- ϛ̅ κα]ὶ δάκτυλοι ϛ̅

10 Cf. Tannery 1881, esp. p. 158 and 163, and Heiberg's attempt to make sense of the corrupted passage Did. 23-24 (Heiberg), which is reflected in his emendations and translation.



(Problem 2) [-ca.?-] . . [-ca.?-] . τετράγονον μίουρον τὸ μηκ( ) [-ca.?-] [-ca.?-]ν̣ τριγόνων δακτύλων λϛ κάθετο̣[-ca.?-] [-ca.?-] . . τριγόνων δακτύλων λϛ ω . . . [-ca.?-] [-ca.?-] . . . . θε . . . . ε̣ γείνονται̣ [-ca.?-]
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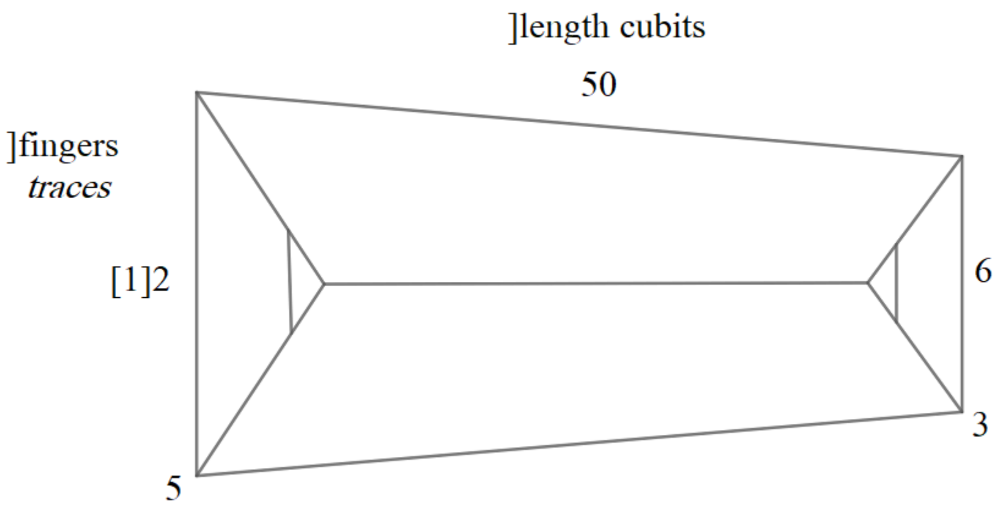
1 *l.* χυδαῖοι Αωχ. οἱ δὲ οιδ . [-ca.?-] prev. ed. 2 [-ca.?-] . πη prev. ed. 3 [-ca.?-]δάκτυλοι prev. ed. 4 *l.* τετράγωνον τὸ μὴ κ[-ca.?-] prev. ed. 5 *or* κάθετο[ς (?) -ca.?-] κάθεσ̣[-ca.?-] prev. ed. 6 *l.* τρίγωνον (*or* τριγώνων) ὡς σ . [-ca.?-] prev. ed. 7 [-ca.?-] . . αι θε prev. ed.

(Problem 1) … The result is 1,800 “mixed” fingers. Divide it by 288, the remainder by 12 in order to convert to [cubits? 6] and fingers 6.



(Problem 2) … tapering quadrilateral with the length of … triangle(s?) 36 fingers, height … triangle(s?) 36 fingers … result …

§19 2-3ἵνα [ϛ̅]: ἵνα was probably followed by γένωνται or ὦσι; if the latter, the word πήχεις may have been included with the first numeral. There is probably not enough room in the lost beginning of line 3 for πήχεις στερεοί. The restored number of (solid) cubits is guaranteed by the preserved number of "mixed" fingers in l. 1, the conversion factors in l. 2, and the number of (solid) fingers in l. 3.

§20 Diagram[ι]β: it is difficult to see anything to the left of what seems to be a beta, but there might be a fold in the papyrus there.

§21 4-7The content of the second problem on the papyrus is irrecoverable. One may wonder whether a "tapering quadrangle" means a trapeze, in which case the task of the problem may have been to find its surface area, possibly similarly to Problem 3 of P.Ayer (= P.Chic. 3, col. 2 1-2; 1st-2nd c., Hawara? TM 63301) or problem a5 in P.Math. But too little text survives to allow for a reconstruction.

5. Ptolemaic, Nicomedian, and solid cubits in P.Oxy. 49 3455 and mensuration of wood

§22 A poorly preserved papyrus, P.Oxy. 49 3455 (3rd-4th c.; TM 64339), contains several metrological texts, one of which lists three measures referred to as "the so-called Ptolemaic cubit," a "Nicomedian cubit," and a "solid cubit" (lines 4-20). Each of the "cubits" is first described by its linear dimensions of length in cubits and width and thickness in fingers. This is followed by the volume, which is computed as the product of the three dimensions, cf. l. 8: τὰ μέτρα πολυπλασιασθέντα "the dimensions multiplied together," and recorded in χυδαῖοι δάκτυλοι, "mixed" fingers, that is 1 sq. finger × 1 cubit units. Finally, the corresponding number of ἀγελαῖοι δάκτυλοι, "ordinary" fingers is given, which is 24 times the number of "mixed" fingers, that is, it results from converting 1 cubit of the "length" of the "mixed" fingers into 24 fingers. Consequently, ἀγελαῖοι δάκτυλοι simply equal cubic fingers, and presumably it is to this equivalence that they owe their name of "ordinary" fingers.[11]

§23 The so-called Ptolemaic cubit amounts in volume to one-half of a cubic cubit and has a thickness of 12 fingers (= 3 palms); it can thus be identified with the 3-palm solid cubit in P.Math. and P.Harris 50.

11 The identification of ἀγελαῖοι δάκτυλοι with cubic fingers finds further confirmation in pseudo-Heron's *Geometrica* 23.67 (Heiberg).

The Nicomedian cubit amounts to one-third a cubic cubit and is 8 fingers (= 2 palms) in thickness; it is thus identical with the 2-palm solid cubit postulated by Tannery and attested in Didymus and P.Math. The following table summarizes the dimensions recorded in the papyrus:[12]

The name of the cubit in P.Oxy. 49 3455	μῆκος Length	πλάτος Width	πάχος Thickness	χυδαῖοι δάκτυλοι “mixed” fingers	ἀγελαῖοι δάκτυλοι “ordinary” fingers
Πτολεμαικός Ptolemaic	1 cubit	24 fingers	<12> fingers	288	[6,]912
Νικομηδικός Nicomedian	1 cubit	24 fingers	8 fingers	[1]92	[4,608]
στερεός Solid	1 cubit	24 fingers	24 fingers	[576]	13,824

§24 The units in which linear dimensions of the three cubits are given follow the same pattern as the descriptions of the solids in mensurational problems, in which volume is expressed in 2- and 3-palm solid cubits. The pattern suggests that objects so described and measured are significantly larger in length than in width or thickness, and that measuring them was a common enough procedure to warrant peculiar metrological units. The commodity that fits best is wood or timber, which is precisely what Tannery argued the 2-palm solid cubit was used for. Tannery, however, had to deal with only one peculiar cubit, while the evidence available now indicates that there were at least two different solid cubits not equal in size to a cubic cubit, which were likely used to measure wood. And wood could apparently be measured in “regular” cubic cubits, too.

§25 If three different measures were used for the volume of wood or timber, the choice which one to use perhaps depended on the type of wood or the cut of timber, while the name of the measure may have referred to the geographical area with which it was for some reason connected. The designation “the so-called Ptolemaic cubit” for the unit equaling one-half of a cubic cubit should presumably point to its Egyptian origin and/or usage. One can compare the qualifier “Ptolemaic” or “Egyptian” applied to the foot in order to distinguish it from the “Roman”, or “Italian”, foot (cf. Did. 9-10 Heiberg; P.Math. G recto 11-13), with the linear Roman foot equaling 13 1/3 fingers, as opposed to the Ptolemaic foot of 16 fingers.[13] Why, however, a volumetric unit characterized by one of its linear dimensions, which was 12 fingers = 3 palms and equal to the Egyptian unit “small span” or Greek *spithame*, should be specifically associated with Egypt is not immediately clear.

§26 We might be on slightly firmer ground with the term “Nicomedian cubit.” The province of Bithynia, of which Nicomedia was the capital, was exceptionally rich in wood. Timber, including fine ship-timber, was one of the main commercial commodities of Nicomedia throughout its history, and the city was famed for its woodworkers and shipbuilders.[14] An association of a volumetric unit in which wood was measured with Nicomedia is thus not surprising, and it is tempting to conjecture that the unit may have been specifically used for ship-timber. Curiously, the papyrus detailed the commodity, which was “purchased” (?) in Nicomedian cubits, but the word for it is unfortunately lost at the beginning of line 11. Yet, the added clarification might indicate that the measure was less common in Egypt than the 3-palm Ptolemaic solid cubit, for which we have evidence in three problems in P.Math. (a3, c1 and g4) and in P.Harris 50.

12 For details on the readings of the figures, see the reedition of the text below.

13 In pseudo-Heron’s *Geometrica* the foot of 16 fingers is referred to as Philetaric (Φιλεταίρειος) or royal (βασιλικός).

14 See Meiggs 1982: 357, 393; Robert 1978, with discussion of the wood and timber industry around Nicomedia in connection with two epitaphs from the city commemorating a wood-carver, ξυλογλύφος (SEG 28 1037), and a rafter, σχεδιοναύτης (SEG 28 1040), who, Robert argues, must have made his living by transporting wood by raft, p. 413-428.

§27 The use of the regular cubic cubit comprised of 24 fingers has not so far been encountered in mensurational problems, but is possibly attested in P.Köln 1 53 (AD 263, Antinoopolis; TM 15464). The papyrus contains an account presented to the council of Antinoopolis regarding the acquisition of wooden beams (ξύλα) for a ceiling of a gymnasium. The account lists the linear dimensions and volumes of pine and fir beams, with the length in cubits, the width and thickness in fingers, and the volume in solid cubits and fingers, πήχ(εις) στερ(εοί) and δάκτυλοι. Unfortunately, for no beam are more than two dimensions preserved, making it impossible to determine the parameters of the unit designated as πῆχυς στερεός. The editor of the papyrus, Robert Hübner, considers the choice between a solid cubit corresponding to a cubic cubit and one corresponding to the 2-palm solid cubit postulated by Tannery and which I identify with the Nicomedian cubit of P.Oxy. 49 3455. Weighing the likelihood of different lengths of the beams derived from the application of either solid cubit to express volume, he concludes that the solid cubit equal to a cubic cubit is a likelier candidate for the ceiling beams in the papyrus.[15] If so, this might give some indirect support to the supposition that the smaller, aka Nicomedian, solid cubit, was used for measuring ship-timber, which would presumably be particularly valuable. Whether the so-called Ptolemaic cubit could then be the Egyptian counterpart of the Nicomedian cubit or a measure used for yet another type of wood or timber remains a question.

§28 J. Shelton, who published P.Oxy. 49 3455, suggested that the section reedited below provided dimensions for (1) a so-called Ptolemaic, or Egyptian, *chous*, with a size of half a cubic cubit, (2) a Nicomedian measure, which he somewhat hesitantly identified with the *kotyle*, and (3) a cubic cubit. Since the third unit discussed in the papyrus is a regular cubic cubit, it is likely that the passage enumerates three volumetric units identified as different kinds of cubits, with the first two equaling the 3- and 2-palm solid cubits in P.Math. and elsewhere. The reedition of the papyrus is based on the image available in Oxyrhynchus Papyri Online.[16]

[ὁ κα]-
λούμενος Πτολε̣[μαικὸς πῆχυς ἔχι τὸ μὲν μῆκος]
πῆχ̣(υν) ἕ̅ν̅α̅, τὸ δὲ πλάτος δακτύ̣[λω]ν̣ κ̅δ̅,
τ̣ὸ δὲ πάχ[ο]ς δακτύλων κ̅δ̅, ὡ̣ς τὰ μέτρ̣α
πολυπλασιασθέ̣ν̣ταγ̣ εἶναι τὸν̣ [π]ῆ̣χοις χυδέ-
[ων] μ̣ὲν δακτύλων σ̅π̅η̣̅, ἀ̣[γ]ε̣λέων
[δὲ S]⳨ιβ. ὁ δὲ Νικομηδικὸ̣ς̣ [πῆχ]ις ἐν ᾧ̣ τὰ
[̣ ̣ ̣ ̣ ̣ ̣ ̣] ̣ α̣ ὠ̣νῖται [ἔχι] τ̣ὸ μ̣ὲ[ν μῆκ]ο̣ς πῆχυν ἕ̅ν̅α̅,
[τὸ δὲ πλ]άτος δακτύλων [κ̅δ̅] τ̣ὸ̣ δ̣ὲ̣ πά-
[χος δα]κτύλων η̣̅, ὡ̣ς εἶν̣[αι τὸ]ν Νικο-
[μηδ]ι̣κ̣ὸν πῆχυν̣ χοιδέ̣ω̣ν̣ μ̣ὲ̣ν δακτύ
[λων ρ]ο̣̅β̅, ἀγελέων δὲ [Δ]χ̣̅[η̅]
[ὁ δὲ στ]ερεὸς πῆχ̣οις ἔχι τὸ μ̣ὲ̣ν̣ μῆκος
[πῆχυ]ν̣ ἐ̣[ν̅]α̣̅, τ̣ὸ̣ δ̣ὲ̣ πλάτος̣ δακτύ̣λω̣ν κ̅δ̅,
[τὸ δὲ πά]χος δακτύλων κ̣̅δ̣̅, ὡς ε̣ἶναι τὸν

15 P.Köln 1 53, p. 149-150.

16 https://doi.org/10.25446/oxford.21168457

[στερε]ὸ̣ν̣ π̣ῆ̣χ̣υ̣ν̣ χ̣[υδ]έ̣ων μὲν δακτ̣ύ̣λω̣ν̣
[φ̅ο̅ς̅,] [ἀγ]ελέων δὲ (μυριάδα) α Γω̅κ̅δ̅

5 χοῦς prev. ed. *l.* ἔχει **7** *l.* δακτύλων ι̅β̅ **8** *l.* πῆχυν τοὺν χόις prev. ed. **8-9** *l.* [ων] **9** *l.* ἀγελαίων **10** *l.* πῆχυς [̣ ̣]ις prev. ed. **11** *l.* ὠνεῖται *l.* ἔχει **14** *l.* χυδαίων **15** *l.* ἀγελαίων **16** *l.* πῆχυς *l.* ἔχει **19** *l.* χυδαίων **20** *l.* ἀγελαίων

The so-called Ptolemaic cubit has the length of one cubit, the width of 24 and the thickness of 24 (sic! understand 12) fingers, so that the dimensions multiplied result in a cubit measuring 288 “mixed” fingers, 6,912 “ordinary” ones. The Nicomedian cubit, in which … are purchased (?) has the length of one cubit, the width of [24] fingers, the thickness of 8 fingers, so that the Nicomedian cubit measures [1]92 “mixed” fingers, [4,608] “ordinary” ones. The solid cubit has the length of one cubit, the width of 24 fingers, and the thickness of 24 fingers, so that the solid cubit measures 576 “mixed” fingers, 13,824 “ordinary” ones.

§29 5Πτολε̣[μαικὸς πῆχυς: the ed.pr. restores χοῦς on the ground that πῆχυς is a unit of length and not of capacity, cf. ln. 10, and because he reads τοὺ̣ν χόις (l. τὸν χοῦν) in l. 8, for which see the comm. below. There is, however, plentiful evidence for πῆχυς used as a unit of capacity, cf., for example, Did. 7 (Heiberg); P.Chester Beatty Codex AC 1390 (c. 275-350, Upper Egypt; TM 61614), 1.11, 2.6. Furthermore, since χοῦς is a liquid measure of capacity, it is unlikely that it would be described as a rectangular prism, which is how all three cubits are presented in the list.

§30 7πάχ[ο]ς δακτύλων κ̅δ̅ (l. ι̅β̅): Shelton rightly noted that the number of fingers in this unit, which is given in ll. 8-9, makes it certain that one of the dimensions recorded as δακτύλων κ̅δ̅, either for τὸ πλάτος or τὸ πάχος, must be a mistake for δακτύλων ι̅β̅. Since it is τὸ πάχος, “thickness”, in l. 13 that has 8 fingers, I suspect that by analogy it was τὸ πάχος here, too, that was supposed to have the unusual and thus defining number of 12 fingers (the usual being 24).

§31 7-8ὡς τὰ μέτρα πολυπλασιασθέντα: the linear dimensions, which are given in cubits for length and in fingers for width and thickness, are multiplied without being converted to the same unit.

§32 8-10The volume of the measure is expressed in χυδαῖοι δάκτυλοι, “mixed” fingers, which are then converted to ἀγελαῖοι δάκτυλοι, “ordinary” fingers. For the terms, cf. §§ 16–17 and 22 above. Following a tentative suggestion of F. Hultsch (*Metrologicorum scriptorium reliquiae* [Leipzig 1864], vol. 1 p. 37 fn. 2) Shelton correctly interpreted ἀγελαῖοι δάκτυλοι as cubic fingers, but his interpretation of χυδαῖοι as square fingers is misleading. He appears to have assumed that χυδαῖοι δάκτυλοι refer to the surface area of the face of the measure formed by its length (μῆκος) and width (πλάτος), 1 cubit (= 24 fingers) × 12 fingers = 288 sq. fingers. However, since the units designate the product of three linear dimensions (ὡς τὰ μέτρα πολυπλασιασθέντα), they must be volumetric, i.e. 288 sq. finger × cubit.

§33 8τὸ̣ν̣ [π]ῆχοις: the ed.pr. has τοὺ̣ν χόις, but the traces before χοις are actually more compatible with eta than with nu, while what Shelton took as a trace of upsilon is likely to be that of nu. Thus, τὸ̣ν̣ [π]ῆχοις, l. τὸν πῆχυν, fits the traces better.

§34 10Νικομηδικὸ̣ς̣ [πῆχ]ις: since the unit is called the Νικο[μηδ]ι̣κ̣ὸν πῆχυ̣ν̣ in ll. 13-14, [πῆχ]ις is likely to be restored here for πῆχυς. Shelton expressed concern that there might not be enough room for more than two letters, but the spacing of the writing, let alone the state of preservation of the papyrus, do not allow for such precise estimations here. Three letters seem perfectly possible.

§35 11[......] α̣ ὠ̣νῖται The word indicating what was purchased (?) in Nicomedian cubits is unfortunately lost, and the reading ὠ̣νῖται itself is far from certain.

§36 12[τὸ δὲ πλ]άτος δακτύλων [κ̅δ̅]: the restoration of the numeral, 24, depends on the number of χυδαῖοι, or "mixed" fingers, in this unit in l. 15, which clearly ends in -2 and which I restore as [ρ]ϙ̣β (since 1 cubit × 24 fingers × 8 fingers = 192 "mixed" fingers).

§37 12-13[κ̅δ̅] τ̣ὸ̣ δ̣ὲ̣ πά|[χος δα]κτύλων: although the traces before the very clearly written πά- are quite faint, I believe they can be reconciled with the suggested – and expected – reading. In the ed. pr. Shelton printed [..]..ος πά|[χος δὲ δα]κτύλων in the text and suggested in the notes that [τὸ δὲ πλ]άτος δακτύλων [γ̅, π]ά̣χος {πά|[13][χος} δὲ δα]κτύλων be restored, i.e. with dittography of the word πάχος.

§38 13-15Shelton suggested the following restoration in his notes: ὡ̣ς εἶν̣[αι τὸ]ν Νικο|[14][μηδ]ι̣κ̣ὸν πῆχυν̣ (l. χοῦν) χοιδέῳν̣ μ̣ὲν δακτύ|[15][λων] ο̣̅β̅, ἀγελέων δὲ [φ̅ο̅ϛ̅], "so that the Nicomedian chous (?) has 72 square dactyls, 576 cubic ones."

§39 15[ρ̅]ϙ̣̅β̅: despite Shelton's statement that the traces before beta fit best an omicron and that koppa is paleographically unlikely, I believe that the circle visible on the papyrus belongs to a koppa and thus restore the numeral accordingly.

§40 19-20The completely preserved figure of 13,824 for the number of ἀγελαῖοι δάκτυλοι in which a cubic cubit is measured (l. 20, ἀγ]ελέων δὲ (μυριάδα) α̅ ͵Γωκδ) proves that they are equivalent to cubic fingers.

Conclusion

§41 Evidence from Didymus and the papyri suggests a trade-specific system for mensuration of wood and timber. Although the logs and pieces of timber were considered geometric solids and usual algorithms were applied to compute the volume, there were certain conventions that distinguished measuring wood from other objects. Likely for practical reasons the length of wood or timber was measured in cubits, but the dimensions of the cross-section in fingers, with no effort spent on converting the different units to one. Secondly, the volume was computed as the product of unconverted linear dimensions so that the result was in notional mixed units equaling 1 sq. finger × 1 cubit, which were sometimes identified as χυδαῖοι δάκτυλοι, "mixed" fingers. These served as a means of computation and were not volumetric units proper. To express the volume, the product in "mixed" fingers was to be converted to one of three measures, all regarded as solid cubits, but varying in size and equaling one-third of a cubic cubit (2-palm solid cubit), one-half of it (3-palm solid cubit), or a cubic cubit. The choice of the unit must have been obvious enough not to warrant explicit specification in the texts of the problems we have. I am inclined to think that it depended on the type and/or cut of wood, possibly with ship-timber measured in the smaller unit(s).

§42 Regardless of the volume of a particular solid cubit, it was divisible into 24 fingers. These fingers were consequently relational and not absolute subunits. The expediency of the ratio of linear subunits (1/24) for the rate of volumetric subunits is obvious, since the regular cubic fingers would have been too minute a measure to be practical in cutting and measuring wood (the relations would be 1/4,608, 1/6,912, and 1/13,824, respectively, for the three solid cubits to cubic fingers).[17] The corollary of expressing volume with the help of relational solid fingers is that mensuration of wood entailed use of

17 Cf. the discussion of the volumes of two beams in P.Köln 1 53, which are recorded as 3 cubits 6 fingers (line 11) and 3 cubits 1 finger (line 17). The small number of fingers makes it certain that they are relational and not cubic fingers, for, in the words of the papyrus's editor, "im Verhältnis zu 1 Elle^3 = 13824 Finger^3 wären die Angaben 6 Finger und 1 Finger unsinnig klein, wenn es sich um echte Kubikfinger handelte," P.Köln 1, p. 147.

entirely different entities all called “fingers,” and that was on top of a range of solid cubits of different sizes. The system would have surely look confusing to an external observer, and it is perhaps for this reason that the great Egyptian mathematician Abu Kamil (ca. 850-ca. 930) criticized “people of Egypt” for their way of measuring wood in his work *Book on Mensuration*.[18] Although the details of the critique are not entirely clear, the major point seems to be that measuring wood should not differ from that of any other solids, indicating that in practice it was. In particular, Abu Kamil notes that “people of Egypt, for measuring their wood, proceed according to something … which is a measurement neither of volume nor of surface.” One wonders whether this reflects his disapproval of computing the volume in notional “mixed” units and then converting it into solid cubits of different dimensions, the subunits of which have the ratio of linear fingers to cubits, all of which is unnecessary from a mathematical point of view.

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18 Measuring of wood is the subject of chapter 34. For an edition with translation, cf. Sesiano 2014: 359–408. I am grateful to Jacques Sesiano for bringing this passage to my attention.