# New Light on P.Mich. 15686 

C. Michael Sampson

Heidelberg: Propylaeum, 2022

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As ever, the Papyrology Collection at the University of Michigan facilitated my research: at my request, head of conservation Marieka Kaye and collection manager Monica Tsuneishi promptly arranged for the papyrus both to be remounted and newly photographed. I am also grateful Terry Wilfong and Michelle Fontenot at the Kelsey Museum of Archaeology, to Julia Lougovaya for meticulous feedback, to Mae Sawatzky both for her careful digital transcription of the editio princeps and for questioning the papyrus' arithmetic on several occasions, and to my collaborator Drew Wilburn for his deep knowledge of the archaeology of Karanis. Financial support for our work was provided by the Social Sciences and Humanities Research Council of Canada as part of the research project t' the Books of Karanis' and by the Faculty of Arts at the University of Manitoba. Any errors are mine.

## Introduction and Archaeological Context

§1 P.Mich. inv. 5663a preserves dozens of individual papyrus fragments which have been reassembled into several mathematical tables, including both division and multiplication. ${ }^{1}$ It was originally published by Sijpesteijn as $\mathbb{C}$ P.Mich. 15686 , but in the process of transcribing and reviewing the text for $\checkmark$ © papyri.info, several deficiencies in the editio princeps came to light. For one, Sijpesteijn declined to reconstruct the columns of the principal fragment in their entirety, even though enough of its multiplication table survives to establish the layout. For another, the edition contains a few errors. ${ }^{2}$ Most problematically, he also left portions of the text untranscribed (no doubt in error). ${ }^{3}$ Because my edition not only improves the various deficiencies of the editio princeps but also both restores the papyrus as fully as possible and renumbers the lines throughout, I have not noted its various divergences from Sijpesteijn's text.
\$2 In the course of reassessing the papyrus, I not only inspected it in person, but was also aided by infrared images provided by the Papyrology Collection at the University of Michigan. These images have both improved the readings and determined the position of several smaller fragments: only two groups of fragments from the papyrus' second and third exercises remain unplaced. Apart from a few places there, the organization and contents of the papyrus are not in doubt.
§3 Michigan's © APIS catalogue includes forty-four separate items under inventory number 5663, but that number does not reflect the ongoing efforts of the collection's conservators. Including 5663a, there are sixty-three distinct groups of fragments. ${ }^{4}$ I observed no ink on the fragments in twelve of the groups. All were unearthed at Karanis during the University of Michigan's 1929 excavation season and were assigned the field number 29-B163*-J. ${ }^{5}$ House B163 is located on the eastern side of the village, lying at the intersection of four squares in the site-wide triangulation grid ( $\mathrm{G}-\mathrm{H}, 10-11$ ) (Figure A). It is noteworthy for its location at the edge of the escarpment - "in some places thirty or forty feet high" ${ }^{6}$ - carved by sebakh-diggers, who mined the central part of the kôm to bedrock before the

[^0]Michigan excavations commenced in 1924, completely destroying a substantial part of the ancient site. ${ }^{7}$ Archival images in the Kelsey Museum of Archaeology capture the extent of the destruction and how close house B163 came to the precipice: in Figures B-C, the walls of its topmost occupation level (i.e., House 125) visibly extrude along the edge of the escarpment. The mud bricks from its collapsed sections litter the slope below.

[^1]

Figure A: Location of B163. Maps and Plans. Map No. 11. Key Plan. E-K, 9-12, Second Layer (B), Eastern Area (Kelsey Museum of Archaeology, University of Michigan, neg. no. 4.2970).


Figure B: A view showing 122, 124, 125, 126, 127 and 129 as they stand on the dump head (Kelsey Museum of Archaeology, University of Michigan, neg. no. 5.3041).


Figure C: A closer view of 122H, showing also walls of 124, 125 and 127 (Kelsey Museum of Archaeology, University of Michigan, neg. no. 5.3040).
\$4 The archaeology of B163 is complicated by several interrelated factors. The first is the documented disturbance of Karanis' archaeological record, including digging at the kôm from as early as $1890^{8}$ and the aforementioned ravaging by sebakh-diggers, which continued in concert with the excavations. The second is the stratigraphy in this area of Karanis, which upon closer analysis is rather unclear (see further, below). The third is the fact that the progress of excavation stalled. Work began on the uppermost occupation level, including House 125, during the initial 1924 season, but was then interrupted for several years as the team focused its efforts elsewhere. The excavation of the lower levels, including House B163 / C163, did not resume until the 1928 season, advancing further in the subsequent 1929 season. ${ }^{9}$ The result of these various realities is that excavation data need to be interrogated carefully.

The basic outline of structures and occupation levels is clear. The house was located at what appears to be the corner of an insula of buildings, bordered by streets to both the east and the south. In all three occupation levels (sc., A, B, and C), these streets follow the same course (Street $112=\mathrm{BS} 165=$ CS220; Street $126=\mathrm{BS} 170=\mathrm{CS} 220) .{ }^{10}$ So too is the footprint of the home's exterior walls consistent across the three levels: with the exception of House 125 's western wall, they follow those of the structure B163 / C163 below it (Figure D). Given that we are dealing with the same structure in all three levels, house 125 was evidently renumbered in later seasons after excavation had revealed more of it. ${ }^{11}$ Outside, there is a small but discernible gap between the top two layers: Street 112 , for example, lies slightly more than a meter above BS170 below. That is larger than the gap between the same levels in areas immediately to the south, but not as considerable as the four and a half meter gap between C and D levels. ${ }^{12}$

[^2]

Figure D: Location of 125. Maps and Plans. Map No. 13. Key Plan. E-K, 9-12, Top Layer (A), Eastern Area, E-K (Kelsey Museum of Archaeology, University of Michigan, neg. no. 4.2971).
§6 The relationship of B and C levels is a complicated matter. The excavators themselves admitted regarding this area of the site that "the B and C periods were not so clearly differentiated and it was
at times difficult to decide whether an occupation level in a given house should be assigned to the late C or early B periods." ${ }^{13}$ Like many others in Karanis, House B163 / C163 was occupied for some time, with the basic structure undergoing renovation and modification gradually. The building's outer walls were certainly reused, ${ }^{14}$ but its delapidation limited further conclusions regarding the structure: "Doubtless more of the original construction did survive into the later period, but all evidences of that had been cleared away by the diggers for fertilizer." ${ }^{15}$ The evidence for both C163 as well as its modification and reuse in B163, in other words, is slim.
\$7 Architectural modifications were a necessary response to life in the desert, where the natural accumulation of sand and debris led to a gradual rise in street levels. In time, the construction of additional stories as well as modifications to doors and walls were necessary to ensure the continuing viability of a structure (not to mention means of egress!), as a consequence of which lower levels of buildings fell out of use or became inaccessible. But while the Michigan team observed and documented such alterations at length elsewhere in Karanis, in the case of B163 evidence did not survive sufficiently to permit analysis. It is interesting in this respect that the Field Director's report refers to the sebakhin's activities vis-à-vis "the original construction". The implication is that, after House 125 had been exposed in 1924, the levels below - i.e., B163 / C163 - were further disturbed before excavations resumed in 1928. Because sebakh-diggers continued to work during the months the excavation was dormant, the risk to the archaeological record on that front is clear. ${ }^{16}$
§8 Excavation data belies a clear distinction between C and B levels of occupation. The former does not appear at all in Figure E, a sectional map of the area, which suggests that the decision to distinguish B and C levels came late or was retroactive. ${ }^{17}$ Additional support for that inference comes from the fact that no items are assigned to C163 in Michigan's record of excavated objects, despite a considerable quantity of items that were designated with an asterisk to indicate a findspot below the floor. ${ }^{18}$ During the 1928 season, for example, ten items were discovered in B163, and an additional twenty-four are noted with an asterisk. Of the items excavated in the next season, a further fifty-five bear this designation. Archival photographs in the Kelsey Museum of Archaeology capture the stratigraphy and confirm that the asterisks designate discovery below the structure's mud floor: in one, a basket (29-B163*-AI) and sieve ( $29-\mathrm{B} 163^{*}-\mathrm{Z}$ ) are documented in situ below a layer containing visible protrusions of straw - undoubtedly the house floor (Figures F-G). Although Peterson’s unpublished field report elsewhere reports that "much of their space [sc. in houses B163 / C163 and B162 / C162] had been devoted to storage purposes in both layers of occupation" (my emphasis), ${ }^{19}$ the combined testimony of the record of objects and archival photographs belies a clear distinction between layers. We may therefore conclude that C163 was distinguished much later, during the analysis of the site, and that the decision to identify another occupation level reflects the excavators' inferences about architectural development across Karanis instead of conclusions prompted immediately by the archaeological data. The picture of the structure and its insula is both incomplete and not derived from a meaningful stratigraphy.

[^3]

Figure E: Maps and Plans. Map No. 88. Sectional Map. E10-H and K10, North to South looking East, Section H10 and K10 (Kelsey Museum of Archaeology, University of Michigan, neg. no. 5.7816).


Figure F: A sieve B163*/Z and a fragment of basket AI as found (Kelsey Museum of Archaeology, University of Michigan, neg. no. 5.3454).


Figure G: Nov. 13, 1929. Stratification of sebbakh beneath B163 (Kelsey Museum of Archaeology, University of Michigan, neg. no. 5.3455).
§9 The nature of the finds from B163 / C163 prompt further questions regarding the archaeology of B163. The excavators concluded that the house "had also been associated with the storage of grain in this part of Karanis", ${ }^{20}$ but one would not draw that conclusion on the basis of excavated objects. The vast majority are small and inconsequential: fragments of glass or lamps, beads, textiles, coins, loom weights, or spindle whorls (see Appendix B for a catalogue.) In the theoretical language of archaeology, they appear to be representative of abandonment or post-abandonment refuse (i.e., trash), which need not reveal anything about the building's systemic context. But even that interpretation is difficult to reconcile with the discovery of so much material below the floor of the structure, an archaeological context most easily interpreted as belonging to the fill predating the structure's construction. The relationship between the excavated objects and the construction of B163 remains nebulous, a problem that perhaps only a ceramic analysis of the pottery assemblage from this context can clarify. ${ }^{21}$

## The Papyrus and Its Contents

\$10 The papyrus itself is dark brown and very brittle. Several of the fragments were reassembled in the field and reinforced with a brown adhesive tape affixed to the verso, at the time of excavation. ${ }^{22}$ The smaller fragments, in particular, are very difficult to read, and infrared photography is be credited not only with facilitating their transcription but also with the further revelation that the text is a palimpsest in at least some places. ${ }^{23}$ The traces of the prior text appear to be the work of the same hand; they are possibly earlier versions of the same arithmetical tables, washed away for whatever reason and recopied. There

[^4]is a kollesis 12 cm from the left edge of the principal fragment, immediately to the left of the sixth column. The backs of all fragments are blank.
§11 The papyrus preserves three distinct tables, which I discuss individually for the sake of clarity. The principal one is a multiplication table in triads which iterates over thirty-six values from two to ten thousand. ${ }^{24}$ The multiplicands are organized in groups by order of magnitude: ones (2-9), ${ }^{25}$ tens ( $10-$ 90 ), hundreds (100-900), and thousands ( $1,000-10,000$ ). For each, the multiplier advances from 1 to 10 , and each group of ten is separated by a paragraphos. Thousands are (as usual) indicated by a hook surmounting numerals between one and nine; myriads by a mu surmounted by the digit. ${ }^{26}$ Across sixteen columns, 360 individual products up to 100,000 are calculated, though the number of lines per column varies widely. With thirty-one lines preserved in full, column seven is the longest. By the end of the multiplication table, contrastingly, as the indication of myriads or a numeral in the thousands effectively doubles each row's height, there are as few as sixteen lines per column. Since as few as seven entries need to be restored in the first column, a sizeable lacuna evidently precedes the table. Its text remains a matter of guesswork. The presence of another exercise cannot be discounted, but as far as the multiplication table goes, the parallels suggest that not much is missing. ${ }^{27}$ The likeliest possibilities include the heading $\pi$ o $\lambda v \pi \lambda \alpha \sigma \iota \alpha \sigma \mu$ ós and a simplied entry for 1 (i.e., ${ }^{\prime \prime} \pi \alpha \xi \alpha \alpha$ ), but these may not have appeared at all.
§12 Sijpesteijn interpreted the second text, preserved on fragments A and B , as a table of division, but quotients are preserved only on fragment B and in many places even the dividend is lacunose. Where the complete calculation is extant, it follows the typical ancient practice, in which the division of a whole number is expressed via a fraction: where we would say, for example, that "fifteen divided by three is five", the ancient formulation is "a third of fifteen is five". But the arrangement of the calculations is noteworthy and unusual: rather than apply multiple divisors to a single dividend in sequence in the usual fashion of a division table, both values change in each entry. In fragment B, where the dividend and divisor are multiples of ten, the formula is (). ${ }^{28} \mathrm{By}$ analogy, one reasonably hypothesizes that the first ten lines of fragment A, where the dividends are single-digit, took the basic form , ${ }^{29}$ although in the absence of both dividend and quotient it is admittedly possible that this table took the usual form and used the same dividend in each entry. The two fragments might only appear to belong together, in other words; their precise placement in either case remains unclear.
§13 One reason for the uncertain organization of the table of division is that both fragment A and B contain additional text that complicates the matter. Below the paragraphos following fragment A.10, for example, there appears to follow a different portion of the same table. Its readings are more challenging and neither dividend nor quotient is preserved, but the formula may well involve hundredths on the same pattern as elsewhere (i.e., ); if so, this section is peculiar for presenting its computations in descending (instead of ascending) order.

[^5]§14 The second column of fragment B , by contrast, seems wholly unrelated to the first. Its text is too threadbare to be certain, but what remains suggests a wholly different table of division, perhaps on the model of the papyrus' third text, preserved on fragments C and $\mathrm{D} .{ }^{30}$ There, a table of division transforms thirtieth parts into their constituent fractions, with the values of the dividend increasing by one half (see Table 1). ${ }^{31}$ In Sijpesteijn's editio princeps, the second column of fragment B belonged to the table transforming thirtieth parts, but my reassessment of his readings makes that organization impossible. Of the four entires, only three are at all legible (and none completely), and they each include the symbol for $2^{\prime}(=1 / 2)$. Two of them transform into a third and a twentieth (in the latter case, an additional fraction has presumably been lost), while the quotient of the last is a third and a tenth. It is tempting to posit, with Sijpesteijn, that these calculations belong to the third text, but the presence of halves in three successive entries belies that hypothesis, as does the appearance of the corresponding entries in fragment C (fr. C.22-25). Other possibilities are that the halves belong to the quotient of an entirely different calculation or that the column preserves a draft or previous version of the table. The extant text, unfortunately, is opaque.

Table 1: The Complete Third Table (Transformations of Thirtieth Parts)

| 1 | $=$ | $30^{\prime}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11/2 | $=$ | $20^{\prime}$ |  |  |  |  |
| 2 | $=$ | $15^{\prime}$ |  |  |  |  |
| $21 / 2$ | $=$ | $15^{\prime}$ | + | $60^{\prime}$ |  |  |
| 3 | $=$ | $10^{\prime}$ |  |  |  |  |
| $31 / 2$ | $=$ | $10^{\prime}$ | + | $60^{\prime}$ |  |  |
| 4 | $=$ | $10^{\prime}$ | + | $30^{\prime}$ |  |  |
| $41 / 2$ | $=$ | $10^{\prime}$ | + | $20^{\prime}$ |  |  |
| 5 | $=$ | $6^{\prime}$ |  |  |  |  |
| $51 / 2$ | $=$ | $6^{\prime}$ | $+$ | $60^{\prime}$ |  |  |
| 6 | $=$ | 5 |  |  |  |  |
| $61 / 2$ | $=$ | $5^{\prime}$ | $+$ | 60' |  |  |
| 7 | $=$ | $5^{\prime}$ | $+$ | $30^{\prime}$ |  |  |
| $71 / 2$ | $=$ | $4^{\prime}$ |  |  |  |  |
| 8 | $=$ | $4^{\prime}$ | + | $60^{\prime}$ |  |  |
| $81 / 2$ | $=$ | $4^{\prime}$ | + | $30^{\prime}$ |  |  |
| 9 | $=$ | $4^{\prime}$ | + | $20^{\prime}$ |  |  |
| $91 / 2$ | $=$ | $4^{\prime}$ | + | 15' |  |  |
| 10 | $=$ | $3^{\prime}$ |  |  |  |  |
| $101 / 2$ | $=$ | $3^{\prime}$ | $+$ | $60^{\prime}$ |  |  |
| 11 | $=$ | $3{ }^{\prime}$ | $+$ | $30^{\prime}$ |  |  |
| $111 / 2$ | $=$ | $3^{\prime}$ | + | $20^{\prime}$ |  |  |
| 12 | $=$ | $3^{\prime}$ | + | $15^{\prime}$ |  |  |
| $121 / 2$ | $=$ | $3^{\prime}$ | + | $12^{\prime}$ |  |  |
| 13 | $=$ | $3^{\prime}$ | + | $10^{\prime}$ |  |  |
| $131 / 2$ | $=$ | $3^{\prime}$ | + | $10^{\prime}$ | + | $60^{\prime}$ |
| 14 | $=$ | $3^{\prime}$ | + | $10^{\prime}$ | + | $30^{\prime}$ |
| $141 / 2$ | $=$ | $3^{\prime}$ | + | $10^{\prime}$ | + | $20^{\prime}$ |
| 15 | $=$ | $2^{\prime}$ |  |  |  |  |
| $151 / 2$ | $=$ | $2^{\prime}$ | + | $60^{\prime}$ |  |  |

[^6]| 16 | $=2^{\prime}+30^{\prime}$ |
| ---: | :--- |
| $161 / 2$ | $=2^{\prime}+20^{\prime}$ |
| 17 | $=2^{\prime}+15^{\prime}$ |
| $171 / 2$ | $=2^{\prime}+15^{\prime}+60^{\prime}$ |
| 18 | $=2^{\prime}+15^{\prime}+30^{\prime}$ |
| $181 / 2$ | $=2^{\prime}+10^{\prime}+60^{\prime}$ |
| 19 | $=2^{\prime}+10^{\prime}+30^{\prime}$ |
| $191 / 2$ | $=2^{\prime}+10^{\prime}+20^{\prime}$ |
| 20 | $=3^{\prime \prime}$ |
| $201 / 2$ | $=3^{\prime \prime}+60^{\prime}$ |
| 21 | $=3^{\prime \prime}+30^{\prime}$ |
| $211 / 2$ | $=3^{\prime \prime}+20^{\prime}$ |
| 22 | $=3^{\prime \prime}+15^{\prime}$ |
| $221 / 2$ | $=2^{\prime}+4^{\prime}$ |
| 23 | $=3^{\prime \prime}+10^{\prime}$ |
| $231 / 2$ | $=3^{\prime \prime}+10^{\prime}+60^{\prime}$ |
| 24 | $=3^{\prime \prime}+10^{\prime}+30^{\prime}$ |
| $241 / 2$ | $=3^{\prime \prime}+10^{\prime}+20^{\prime}$ |
| 25 | $=3^{\prime \prime}+6^{\prime}$ |
| $251 / 2$ | $=3^{\prime \prime}+6^{\prime}+60^{\prime}$ |
| 26 | $=2^{\prime}+3^{\prime}+30^{\prime}$ |
| $261 / 2$ | $=2^{\prime}+3^{\prime}+20^{\prime}$ |
| 27 | $=3^{\prime \prime}+5^{\prime}+30^{\prime}$ |
| $271 / 2$ | $=3^{\prime \prime}+5^{\prime}+20^{\prime}$ |
| 28 | $=3^{\prime \prime}+4^{\prime}+60^{\prime}$ |
| $221 / 2$ | $=3^{\prime \prime}+4^{\prime}+30^{\prime}$ |
| 29 | $=3^{\prime \prime}+4^{\prime}+20^{\prime}$ |
| $291 / 2$ | $=3^{\prime \prime}+4^{\prime}+15^{\prime}$ |
| 30 | $=1$ |

§15 Tables of multiplication and division of the sort to which the texts of P.Mich. inv. 5663a belong are not uncommon in the papyri. ${ }^{32}$ That said, both the second one (whose divisors and dividends change in each entry) and the third (transformations of thirtieth parts) are unusual. ${ }^{33}$ Several of the latter's features, in particular, warrant mention. The first is that the divisor (i.e., $30^{\prime}$ ) is nowhere explicitly stated. That is not entirely unexpected: ancient tables of division typically indicate the divisor in a header. But an unfortunate consequence of its absence is that we can only speculate whether it was expressed in fractional form (i.e., $\tau \hat{\jmath} \varsigma \alpha \lambda^{\prime}$ " " 30 ' of one $<$ is 30 " ", cf. © P.Mich. 3 145, II CE) or as an adjective (i.e., $\tau \rho \iota \alpha \kappa o ́ \sigma \tau \iota \alpha$, cf. © TM 64346, III/IV CE). ${ }^{34}$ The grammar is also somewhat unusual: in tables of division, the dividend is normally preceded by an article in the genitive while the divisor (if stated) is preceded by the article in the nominative. So, for example, in the papyrus' second text we read $\tau$ ò $\xi^{\prime} \tau \hat{\omega} \nu$ o $\alpha 5^{\prime}$ (the $60^{\prime}$ of 70 is 1 and $6^{\prime}$, fr. B i.3). In the third, however, an article in the nominative appears despite the divisor's omission. Take the example of fr. D.15: although one expects

[^7]the calculation $\tau \hat{\varrho} v \kappa \angle \omega^{\prime} \xi^{\prime}$（of 20 and $2^{\prime}<a 30^{\prime}>$ is $3^{\prime \prime}$ and $60^{\prime}$ ），it is instead expressed as $\tau \alpha ̀ \kappa \angle \omega^{\prime} \xi^{\prime}$［20 and $2^{\prime}<$ parts（of 30 ）＞are $3^{\prime \prime}$ and 60 ］．${ }^{35}$ Similarly unusual is the one－half increment by which this table progresses through its thirtieth，for which I have found no parallel．
§16 Although they are not abundant，scribal errors on the papyrus are consistent with copying from a model， which implies a textual tradition of one sort as opposed to the actual computation of the values by the scribe，as part of a practical exercise．${ }^{36}$ The text was copied，in other words，as a reference tool．Some of the slips are innocuous．The scribe noted and corrected，for example，several mistakes of arithmetic by overwriting the original product with the correct number（e．g．，ii．23，v．108，vii．143，xi．273，xv．336）． But others went unnoticed（e．g．，xi．269，perhaps xiii．308，xv．336），the most significant of which appear in three successive lines towards the end of the multiplication table（xvi．347－349）．The nature of the errors here，in which the product as written in each line belongs rather to the previous entry，is more indicative of a jump－either in the model or in the scribe＇s copying from it－than of a computational error in several consecutive lines．A product in one of the preceding lines was evidently skipped，and the impact of that mistake was felt until the calculations for 9,000 were completed，at which point the arithmetic resets．

## Palaeography and Date

\＄17 The speed and care with which the hand writes varies．In the multiplication table，at least，it is careful but fluent：the capitals are well－formed，more often spaced generously than not，and roughly bilinear （gamma，stigma，pi，and eta are occasionally rather squat）．The scribe frequently lifts the pen：stigma and epsilon，for example，are most frequently drawn in two distinct strokes，with the second stroke a rather wide horizontal．In alpha，lifting the pen once allows for a slightly bowed right diagonal atop an angular bubble．${ }^{37}$ Delta is similarly drawn，though its base droops slightly in the manner of a shallow bowl．Other letters tend toward elegant curvature：eta，zeta，mu，nu，pi，and especially both tau and hypsilon all include strokes that are curved，wavy，or gently bowed．The shape of tau is especially noteworthy in this respect for how its crossbar is consistently split，a consequence of being drawn in two strokes．${ }^{38}$ After making the left side of the crossbar in a short，wavy horizontal，the scribe lifts the pen before making the hasta and right side of the crossbar in an ascending stroke that curves clockwise where it meets the left side of the crossbar（see，e．g．，viii．192－199）．The ductus results in a central dip along the crossbar that frequently renders tau quite similar to hypsilon（e．g．，vii．154－155）．Xi and zeta are also easily confused：they are distinguished chiefly by the former＇s diagonal，intersected by a horizontal stroke that is often little more than a subtle blot（e．g．，iv．86；cf．iv．77）．The central element of omega occasionally reclines to the left．The middle element of mu descends deeply and is more angular than rounded．The scribe＇s comfort in drawing rounded letters is indicated by the confident shapes of beta，omicron，and theta，though the smaller loops of koppa and rho are less uniform．Decorative finials frequently adorn the left sides of letters：in addition to the aforementioned shapes with subtle curvature，kappa，chi，and even gamma are slightly hooked．These letters are neither entirely consistent nor regular，but the writing is by no means the work of a novice scribe．${ }^{39}$ The fraction $3^{\prime \prime}(=2 / 3)$ is written as a beta with a tick（cf．the ductus of $2^{\prime}$ ，which is written as $\angle$ ）．${ }^{40}$

35 One should presumably supply $\mu \varepsilon ́ \rho \eta$ as the noun governed by the article $\tau \alpha$ ，though the neuter singular $\tau$ ó is more typical：see $\checkmark$ Fowler（1999：235）．For an overview of ancient division and，particularly，the fractional quotient，see $\longleftarrow$ Azzarello（2018：95－97）and ① Lougovaya（2022：662）．
36 For the former，see e．g．，『 Azzarello 2018，『 © Azzarello 2019；for the latter see 厄 Lougovaya 2022.
37 Admittedly，the bubble becomes less triangular and more round over the course of the table．
38 According to $\begin{array}{r} \\ \\ \text { Cavallo and Maehler（2008：15），＂The tau with split crossbar，the forerunner of the V－shaped tau in documentary }\end{array}$ cursive，does not seem to occur in bookhands before the early second century and can therefore serve as a useful dating criterion．＂
39 Cf．，for example，© P P．Vindob．G 29790 （I CE）．Although similarly adorned with finials，its capitals are drawn more slowly and are more aspirational as regards spacing，bilinearity，and regularity． $\bar{C}$ Cribiore（1996：259，no．346）describes that hand as＂evolving＂， with＂uneven alignment and letter spacing．＂
40 For beta as indicating đò סv́o $\mu \varepsilon ́ \rho \eta$（＂the two parts［sc．＇of three＇］＂），see $\begin{array}{r}\text { T Fowler（1999：228）．}\end{array}$
§18 Sijpesteijn＇s date for the hand（II－III）was presumably based upon inferences about the archaeology of B163．I would assign a decidedly earlier one－in the second or first century BCE，and no later than the first century CE－based on a confluence of considerations．${ }^{41}$ One is the hand＇s predilection for ornamentation in the form of flourishes，finials，and pronounced curvature in many letters．In the cases of pi，eta，and sampi，such curvature，while not unparalleled in Roman hands of the informal round type，${ }^{42}$ is more typical of Ptolemaic hands．The ductus of tau is an especially good indicator： the way that its crossbar is split is best paralleled in Ptolemaic hands，${ }^{43}$ as is the format of indicating myriads，though this last consideration admittedly persists in later eras．${ }^{44}$ Although the script aspires to regularity and rounded bilinearity，${ }^{45}$ the best parallels are early：$\checkmark$ SB 1222 ［ $=\square$ Cribiore（1996： no．249），II－I BCE］is similarly fluent and well－formed，although the sample size of letters in that case is relatively small．In its elegant curvature，ornamentation，and aspirational bilinearity， $\mathbb{C}$ P．Tebt． 3（1） $811(165 \mathrm{BCE})$ is the closest comparison I have found．So too does the hand of $¢ \mathrm{TM} 65784$（＝ $\because$ Cavallo－Maehler（2008：no． 55 ），II BCE）draw many letters similarly，albeit with a more upright， less rounded ductus．${ }^{46}$ The possibility of a relatively early date should not be dismissed out of hand； despite the problems with its archaeology，house B163 is one of the few structures in Karanis that yielded Ptolemaic papyri，and this papyrus was found below its floor．${ }^{47}$ The accumulation of evidence points to the first or second century BCE，though I admit that a date in the first century CE cannot be ruled out．
§19 The other two sections of the text are also the work of the principal hand，but their script is less regular， and decidedly both more compact and rapid．The capitals are less clearly spaced；the ductus of the articles $\tau$ ó and $\tau \hat{\omega} v$ ，in particular，reveals a tendency to ligature the crossbar of tau with the subsequent vowel：see fr．B i． 6 for a particularly good example．

P．Mich．inv．5663a
29－B163＊－J
© https：／／quod．lib．umich．edu／a／apis

| $21($ h $) \times 42($ w $)$ | II－I BCE |
| :--- | ---: |
| 8.1 （h）$\times 8.5($ w） | Karanis |
| $2.2($ h $) \times 2.9$（w） |  |

```
1
i
    [\beta \alpha \beta]
    [\beta\beta\delta]
    [\beta\gamma\varsigma]
    [ }\beta\delta\eta\mathrm{ ]]
5[ [\beta & v]
    [ }\beta\mathrm{ 与^ }\beta\mathrm{ ]
```

41 The parallel he cites，© P．Heid．inv． 1701 （© Seider II 36，III－IV CE）is a poor one，closer to informal capitals or even the severe style． 2 Cf．© P．Lond．Lit． 166 （early II CE）．
43 E．g．，厄P P．Berol．inv． 9772 （匹 Seider II 9，II BCE）；the third hand of® P．Bad． 22 （ 130 BCE ），and © P．Oxy． 121453 （30－29 BCE）．
44 See 『 Fowler（1999：226）：examples of the early format include ¿ ¿P．Cair．Zen． 3 59368．24－26（240 BCE）；『 BGU 81746.10 （64／63 BCE）；『 BGU 162668.3 （ 25 BCE－5 CE）；『 BGU 3 802．8， 19 （42 CE）；『 P．Erl． 93 （II－III）．For later formats，see，e．g．，て BGU 3 976．10－12（II）；『 P．Mich． 11 620v．315－321（240）；『 BGU 184.12 （242／3）．
45 The so－called biblical majuscules of later eras execute aspects of this style in a more restrained and regular way．See，for example，〔 TM 60766 （III／IV CE）．
46 Cf．also in this regard the richly ornamented and upright $\measuredangle$ P．Tebt． 1164 （post 105 BCE）．
47 E．g．，inv．〔 5378 （28－B163A－A）and © 5380 （28－B163A＊－O）．Other examples of Ptolemaic papyri from Karanis include P．Mich．inv． $\longleftarrow 5737$ and $\nearrow 5739$ from house C137．But Roman－era ostraca were also uncovered beneath the floor of B163：e．g．，© O．Mich． 2959 （＝29－B163＊－R，second half II）；© O．Mich 31030 （＝29－B163＊－DI，III）．

$$
\begin{aligned}
& {[\beta \zeta \iota]} \\
& \text { ii } \\
& {\left[\begin{array}{lll}
\beta & \eta & 15
\end{array}\right]} \\
& {\left[\begin{array}{ll}
\beta & \theta][1] \eta
\end{array}\right.} \\
& 10\left[\begin{array}{ll}
\text { r }] ~
\end{array}\right. \\
& {[\gamma \alpha] \gamma} \\
& {[\gamma \beta] \text {. }} \\
& {[\gamma \gamma] \theta} \\
& {\left[\begin{array}{lll}
\gamma & \delta & \beta
\end{array}\right]} \\
& { }^{15}[\gamma \varepsilon][1] \varepsilon \\
& \text { [ } \gamma \varsigma \text { ] [1] } \eta \\
& {[\gamma \zeta][\kappa] \alpha} \\
& \text { [ } \gamma \eta \text { п } \kappa \delta \\
& {[\gamma \theta \kappa \zeta \text { ] }} \\
& 20[\gamma 1 \lambda] \\
& {[\delta \alpha] \delta} \\
& {[\delta \beta] \eta} \\
& {[\delta \gamma] ~ \beta} \\
& \text { [ } \delta \delta] 15 \\
& 25 \text { [ } \delta \varepsilon \kappa \text { ] } \\
& \text { [ } \delta \varsigma][\kappa] \delta \\
& \text { [ } \delta \zeta \kappa \eta \text { ] } \\
& \text { [ } \delta \eta \lambda \beta \text { ] } \\
& \text { [ } \delta \theta \lambda \varsigma] \\
& 30[\delta 1 \mu] \\
& {\left[\begin{array}{c}
\varepsilon \\
\varepsilon
\end{array}\right]} \\
& {\left[\begin{array}{lll}
\varepsilon & 1 & 1
\end{array}\right]} \\
& {\left[\begin{array}{ll}
\varepsilon & \gamma \\
1 & \varepsilon
\end{array}\right]} \\
& \text { [ } \varepsilon \delta \kappa \text { ] }
\end{aligned}
$$

```
35 & [\varepsilon \kappa\varepsilon]
    \varepsilon\varsigma\lambda
    \varepsilon\zeta\lambda\varepsilon
    \varepsilon\eta \mu
    \varepsilon0\mu\varepsilon
40 &\imathv
    5\alpha5
    \varsigma \betaı
    s in
    \varsigma \delta \kappa\delta
45 ऽ \varepsilon\lambda
    \varsigma }\lambda
    \varsigma\zeta\mu\beta
    \varsigma \eta | 
    \varsigma0[v]\delta
50 \varsigma\imath\xi
    \zeta\alpha\zeta
    \zeta 
    \zeta\gamma\kappa\alpha
    \zeta \delta \kappa[\eta]
55[\zeta]\varepsilon[\lambda]\varepsilon
    [\zeta \varsigma] [\mu]\beta
    [\zeta\zeta][\mu]0
    [\zeta\eta v\zeta]
    [\zeta0 %\gamma]
60 [\zeta%o]
    [\eta] \alpha \eta
    [\eta] \beta 15.
iv
    [n] \gamma[\kappa\delta]
```

```
    \eta \delta\lambda\beta
65 \eta \varepsilon 
    \eta }
    \eta\zeta\nu\varsigma
    \eta \xi\delta
    \eta o }
70 \eta I \pi
    0\alpha0
    0 | \eta
    0 }\kappa
    0\delta\lambda\varsigma
75 0\varepsilon \mu\varepsilon
    \vartheta\varsigmav\delta
    0\zeta\xi\gamma
    0 \eta o
    0 0\pi\alpha
80 019
    1\alpha1
    1 \beta\kappa
    l}\gamma
    l }\delta
85 1&V
    1ऽ\xi
    l\zetao
    1\eta\pi
v
    10 %
90 11\rho
    \kappa \alpha \kappa
    \kappa \beta}
```

```
    \kappa }\gamma
    \kappa}\delta
95 \kappa\varepsilon\rho
    \kappa S \rho\kappa
    \kappa}\rho
    \kappa \eta\rho[\xi]
    \kappa}0\rho
100 к \imath \sigma
    \lambda\alpha\lambda
    \lambda\beta\xi
    \lambda
    \lambda\delta\rho\kappa
105\lambda\varepsilon\rhov
    \lambda}\rho\rho
    \lambda\zeta\sigmal
    \lambda\eta\sigma\mu
    \lambda \sigmaо
110\lambda1\tau
    \mu\alpha\mu
    \mu}
    \mu \gamma \rho\kappa
vi
    \mu}\delta\rho
115 \mu\varepsilon\sigma
    \mu \sigma 
    \mu\zeta\sigma\pi
    \mu\eta\tau\kappa
    \mu}\tau
120 \mu v v
    v\alphav
```

```
    v\beta\rho
    v\gamma\rhov
    v \delta \sigma
125v\varepsilon\sigmaV
    v}
    v\zeta\tauv
    v\etav
    v[0]vv
130v! ! !
    -
    \xi\alpha\xi
    \xi \beta \rho\kappa
    \xi}\gamma\rho
    \xi}\delta\sigma
135[\xi\varepsilon] \tau
    \xi}\varsigma\tau
    \xi\zeta טк
    \xi \eta[v]\pi
    \xi0[\varphi]\mu
140 \xi l \chi
vii
    O \alpha O
    o \beta \rho\mu
    o }\gamma
    o }\delta\sigma
145 o \varepsilon\tauv
    O ऽ vK
    o\zetavQ
    o \eta\varphi\xi
    o }0\chi
150 o t %
```

```
\(\pi \alpha \pi\)
\(\pi \beta \rho[\xi]\)
\(\pi \gamma \sigma[\mu]\)
\(\pi \delta \tau[\kappa]\)
\(155 \pi \varepsilon v\)
\(\pi \varsigma v \pi\)
\(\pi \zeta \varphi \xi\)
\(\pi \eta \chi \mu\)
\(\pi \theta \psi \kappa\)
\(160 \pi \mathrm{l} \omega\)
Q \(\alpha\)
Q \(\beta \rho \pi\)
Q \(\gamma\) бO
\(\varphi \delta \tau \xi\)
\(165 \mathrm{Q} \varepsilon v v\)
\(\varphi\) ऽ \(\varphi \mu\)
\(Q \zeta \chi \lambda\)
Q \(\eta \psi \kappa\)
¢ \(\theta[\omega]\)
170 O \(1 \lambda\)
- -
\(\rho \alpha \rho\)
viii
\(\rho \underset{\sim}{\beta}[\sigma]\)
\(\rho \gamma \tau\)
\(\rho \delta v\)
\(175 \rho \varepsilon \varphi\)
\(\rho \varsigma \chi\)
\(\rho \zeta \psi\)
\(\rho \eta \omega\)
\(\rho \theta \lambda\)
\(180 \rho[\mathrm{l}\) A]
```

```
    \sigma\alpha\sigma
    \sigma\betav
    \sigma\gamma\chi
    [\sigma] \delta \omega
185\sigma [\varepsilon] A
    \sigma s A[\sigma]
    \sigma\zeta[A]v
    \sigma \eta A\chi
    \sigma 0 A\omega
190 \sigma i B
    \tau \alpha \tau
    \tau\beta\chi
    \tau }
    \tau \delta A\sigma
195\tau & A\varphi
    \tau \varsigma ~ A \omega
    \tau\zeta B\rho
    \eta Bv
    \tau0B
200 \tau ! \Gamma
    -
ix
    [\begin{array}{lll}{v}&{v}\end{array}]
    [v] \beta\omega
    [v] \gamma A\sigma
    [v] \delta A}
205[v] & B
    [v \varsigma] B[v]
    [v\zetaB\omega]
    [v \eta Г\sigma]
    [v 
```

```
210[lv l A}
```

    \(\left[\begin{array}{lll}\varphi & \alpha\end{array} \varphi\right]\)
    \([\varphi \beta \mathrm{A}]\)
    \([\varphi \gamma \mathrm{A} \varphi]\)
    \(\varphi \delta[\mathrm{B}]\)
    $215 \varphi[\varepsilon \mathrm{~B} \varphi]$
$\varphi \varsigma[\Gamma]$
$\varphi \zeta[\Gamma \varphi]$
$\varphi \eta \Delta$
$\varphi \theta \Delta \varphi$
220 甲 i E
- -
$\chi \propto \chi$
$\chi \beta$ A $\sigma$
$\chi \gamma \mathrm{A} \omega$
$\chi[\delta] \mathrm{Bv}$
$225 \chi \varepsilon \Gamma$
$\chi \subseteq \Gamma \chi$
$\chi \zeta \Delta \sigma$
$\chi \eta \Delta \omega$
X
$\chi \theta \mathrm{E} \delta$
$230 \chi 15$
$\psi \alpha \psi$
$\psi \beta$ Av
$\psi \gamma \mathrm{B} \rho$
$\psi \delta \mathrm{B} \omega$
$235 \psi \varepsilon \Gamma[\varphi]$
$\psi[\varsigma] \Delta[\sigma]$
$\psi \zeta \Delta[\lambda]$
$\psi[\eta \mathrm{E} \chi]$

```
    \psi 0[S\tau]
240 [\psi i Z]
    \omega\alpha\omega
    \omega \beta A\chi
    \omega \gamma Bv
    \omega \delta \Gamma \sigma
245 \omega & 
    \omega s}\Delta
    \omega \zeta \mathrm { E } \chi
    \omega \eta Sv
    \omega0 Z\sigma
250 \omega < H
    - -
\(\lambda \alpha \lambda\)
\(\lambda \beta A \omega\)
\(\lambda \gamma \mathrm{B} \psi\)
\(\lambda \delta \Gamma \chi\)
xi
\(255[\lambda \varepsilon \Delta \varphi]\)
\(\lambda[\varsigma \mathrm{Ev}]\)
\(\lambda[\zeta S \tau]\)
\(\lambda[\eta \mathrm{Z} \sigma]\)
[ \(\lambda \theta \mathrm{H} \rho\) ]
260 [ \(\lambda 1 \Theta]\)
- -
\([\mathrm{A} \alpha \mathrm{A}]\)
[A \(\beta\) B]
[A \(\gamma \Gamma\) ]
[A] \(\delta \Delta\)
\(265 \mathrm{~A} \varepsilon \mathrm{E}\)
A \(\varsigma S\)
A \(\zeta[Z]\)
```

```
    A \eta[H]
    A 0 A
270 A i \\alpha/ \mu(v\rholóc)
    B < [B]
    B \beta \Delta
    B }\gamma\textrm{S
    B \delta H
275 B & \\alpha/ \mu(v\rhoıác)
xii
    [B s \mu(v\rhoı\alphàc) \B/]
    [B \zeta] \mu(vpı\alphà¢)\\Delta
    [B \eta] \mu(vplòc) \S
    [B 0 \mu(v\rhoı\grave{\alpha})\H/]
280 [B \imath \beta/ \mu(v\rhoı\alphá\delta\varepsilonc)]
    [\Gamma \alpha\Gamma]
    \Gamma S
    [\Gamma] \gamma\Theta
    [\Gamma \delta\mu(v\rhoı\alphàc)\B/]
285 [\Gamma \varepsilon \mu(v\rhoı\alphà¢)\E/]
    [\Gamma \varsigma \mu(v\rhoiòc)\H/]
    [Г \zeta \\beta/ \mu(vрı\alphá\delta\varepsilon\varsigma) \A/]
    [\Gamma \eta] [\\beta] \mu(v\rhoı\alphá\delta\varepsilon\varsigma) [\\Delta]
    [Г] 0\\beta/ \mu(v\rhoı\alphá\delta\varepsilon\varsigma) \Z
290[\Gamma] \imath \\gamma/ \mu(vpı\alphá\delta\varepsilon\varsigma)
    [\Delta] & \Delta
    [\Delta] \beta H
    \Delta\gamma\mu(v\rhoıàc)\B
    \Delta\delta\mu(v\rhoı\alphà\varsigma) \S/
295\Delta\varepsilon[\beta/] \mu(v\rho\primeó\delta\varepsilon\varsigma)
xiii
    [\Delta\varsigma\\beta/ \mu(v\rhol\alphá\delta\varepsilonc) \\Delta]
```

$[\Delta \zeta \backslash \beta / \mu(v \rho \iota \alpha ́ \delta \varepsilon \varsigma) \backslash H /]$
$[\Delta \eta \gamma / \mu(v \rho \prime \alpha ́ \delta \varepsilon \varsigma) \backslash B]$
$[\Delta \theta \backslash \gamma / \mu(v \rho \iota \alpha ́ \delta \varepsilon \varsigma) \backslash S /]$
$300[\Delta \mathfrak{i} \backslash / \mu(v \rho \imath \alpha ́ \delta \varepsilon \varsigma)]$
[ $\mathrm{E} \alpha \mathrm{E}]$
$\left[\mathrm{E} \beta \backslash \beta / \mu\left(v \rho\right.\right.$ ló $\left.\left.^{\delta} \varepsilon \varsigma\right) \backslash \mathrm{E} /\right]$
[E $\gamma \mu$ (vpıòc) $\backslash \mathrm{E} /]$
[E $\delta \backslash \beta / \mu(v \rho \prime \alpha ́ \delta \varepsilon \varsigma)]$
${ }^{305}[\mathrm{E} \varepsilon][\beta /] \mu(v \rho \prime \alpha ́ \delta \varepsilon \varsigma)[\mathrm{E} /]$
$[\mathrm{E}] \varsigma[\gamma /] \mu(v \rho 1 \alpha ́ \delta \varepsilon \varsigma)$
$\mathrm{E} \zeta \boldsymbol{\gamma} / \boldsymbol{\mu}(\mathrm{v} \rho \mathrm{o} \delta \delta \varepsilon \varsigma)[\mathrm{E} /]$
E $\eta\left[\begin{array}{l}\delta / \mu(v \rho ı \alpha ́ c)\end{array}\right]$
$\mathrm{E} \theta \backslash \delta / \mu(v \rho 1 \alpha ́ \delta \varepsilon \varsigma)[\mathrm{E} /]$
310 E ı $\backslash \varepsilon / \mu(\nu \rho \neq \alpha ́ \delta \varepsilon \varsigma)$
-ca.?-
5+6+7
xiv
[S $\beta$ ] $\mu$ (vpiòc) $\backslash \mathrm{B} / / /$
[S $\gamma] \mu$ (vрıа̀с) $\backslash \mathrm{H}$
$[S \delta \backslash \beta / \mu(v \rho \iota \alpha ́ \delta \varepsilon \varsigma) \backslash \Delta]$
315 [S ع $\gamma / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma)]$
$[S ~ \varsigma ~ \gamma \gamma / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma) \backslash S /]$
$[\mathrm{S} \zeta \backslash \delta / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma) \backslash \mathrm{B} /]$
[ $\mathrm{S} \eta \backslash \delta / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma) ~ \ H /]$
$[S \theta \backslash \varepsilon / \mu(v \rho 1 \alpha ́ \delta \varepsilon \varsigma) \backslash \Delta]$
320 [S i \ऽ/ $\mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma)]$

-     - 

Z $\alpha$ Z
Z $\beta \mu$ (vpiòc) $\backslash \Delta$
$[\mathrm{Z} \gamma] \backslash \beta /[\mu(v \rho 1 \alpha ́ \delta \varepsilon \varsigma)] \backslash \mathrm{A}$
[ $\mathrm{Z} \delta \backslash \beta / \mu$ (vpıó $\delta \varepsilon \varsigma) \backslash \mathrm{H} /]$
[Z $\varepsilon \backslash \gamma / \mu(v \rho \prime \alpha ́ \delta \varepsilon \varsigma) \backslash \mathrm{E} /]$
$[Z \varsigma \backslash \delta / \mu(v \rho \iota \alpha ́ \delta \varepsilon \varsigma) \backslash B /]$
$[Z \zeta \backslash \delta / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma) \backslash \Theta /]$
$[Z \eta \backslash \varepsilon / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma) \backslash S /]$
xv
$[Z \theta \backslash S / \mu(v \rho \not \alpha ́ \delta \varepsilon \varsigma) \backslash \Gamma]$
$330[\mathrm{Z}$ ı $\backslash / \mu($ vpıó $\delta \varepsilon \varsigma)]$

-     - 

$[\mathrm{H} \alpha] \mathrm{H}$
$[H \beta] \mu$ (vpiòc) $\backslash S$
$[\mathrm{H} \gamma] \backslash \beta / \mu($ vpıó $\delta \varepsilon \varsigma) \backslash \Delta$
$[\mathrm{H}] \delta \gamma / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma)$ B.
$335[\mathrm{H}] \varepsilon \backslash \delta / \mu$ (vрıó $\delta \varepsilon \varsigma)$
H $\varsigma \backslash \delta / \mu(v \rho \prime \alpha ́ \delta \varepsilon \varsigma) \backslash S /$
$[H \zeta] \backslash \varepsilon /[\mu(v \rho 1 \alpha ́ \delta \varepsilon \varsigma)] \backslash S$
$\mathrm{H}[\eta \backslash \varsigma / \mu(v \rho \iota \alpha ́ \delta \varepsilon \varsigma) \backslash \Delta]$
H $[\theta$ \ऽ/ $\mu(v \rho \stackrel{\alpha}{ } \delta \varepsilon \varsigma) \backslash B /]$
${ }^{340} \mathrm{H}[\imath \eta / \mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma)]$
$[\Theta \alpha \Theta]$
$[\Theta \beta \mu(v \rho ı \grave{\alpha} \varsigma) \backslash \mathrm{H} /]$
$[\Theta \gamma \backslash \beta / \mu$ (vрıó $\delta \varepsilon \varsigma) ~ Z /]$
$[\Theta \delta \backslash \gamma / \mu(v \rho \not \propto \alpha ́ \delta \varepsilon \varsigma) \backslash S /]$
xvi
$345[\Theta \varepsilon \backslash \delta / \mu(v \rho 1 \alpha ́ \delta \delta \varepsilon \varsigma) \backslash \mathrm{E} /]$
$[\Theta \varsigma \backslash \varepsilon / \mu(v \rho 1 \alpha ́ \delta \varepsilon \varsigma) \backslash \Delta]$
$[\Theta \zeta]<\langle\varepsilon />\mu(v \rho ı \alpha ́ \delta \varepsilon \varsigma) \backslash \Delta$
$\Theta \eta<\backslash \varsigma />\mu($ vpló́ $\delta \varepsilon \varsigma) \backslash \Gamma /$
$\Theta \theta<\zeta />\mu$ (vpıó $\delta \varepsilon \varsigma)$ B/

$\alpha / \mu$ (vpiòc) $\alpha \backslash \alpha / \mu$ (vpıóc)
$\alpha / \mu$ (vpıòc) $\beta \backslash \beta / \mu$ (vpıó $\delta \varepsilon \varsigma)$

```
    [\\alpha/ \mu(vрı\alphà\varsigma) }\gamma]\\gamma/\mu(\mathrm{ vpló }\delta\varepsilon\varsigma
    [\\alpha/\mu(v\rhoı\alphà\varsigma) \delta] \\delta/ \mu(v\rhoוó\delta\varepsilon\varsigma)
355[\\alpha/ \mu(v\rhoı\alphà\varsigma) \varepsilon \\varepsilon/ \mu(v\rho\iota\alphá\delta\varepsilon\varsigma)]
    [\\alpha/ \mu(v\rhoı\alphà\varsigma) ऽ\\varsigma/ \mu(v\rhoı\alphá\delta\varepsilon\varsigma)]
    [\\alpha/\mu(v\rhoı\alphà\varsigma)\zeta/\zeta/ \mu(v\rhoı\alphá\delta\varepsilon\varsigma)]
    [\\alpha/ \mu(v\rhoı\alphà\varsigma) \eta \\eta/ \mu(v\rhoı\alphá\delta\varepsilon\varsigma)]
    [\\alpha/ \mu(v\rhoı\alphà\varsigma) 0 \0/ \mu(v\rhoı\alphá\delta\varepsilon\varsigma)]
360 [\\alpha/ \mu(v\rhoi\alphà\varsigma) \ \/ / (vpló\delta\varepsilon\varsigma)]
A
    \tauò \alpha[\tau\hat{\}|\beta]
    \tauò <[\tau\hat{v}\gamma\alpha<]
    \tauò [\gamma\tau\omegaि]v[\delta\alpha \gamma]
    \tauò \delta \tau\omegaिv [\varepsilon \alpha \delta]
 \tauò \varepsilon\tau@ิv[5 < &]
    \tauò ऽ \tauติv[\zeta 人 ک]
    \tauò \zeta\tau\hat{v}[\eta\alpha\zeta]
    \tauò \eta \tau\omegaิv[0 \alpha \eta]
```



```
10 \tauò 0 \tau[\hat{vv -ca.?- ]}
    \tauò [-ca.?- ]
    \tauò A [ -ca.?- ]
    \tauò \omega [ -ca.?- ]
    \tauò \psi [ -ca.?- ]
15 ... [-ca.?- ]
B
i
    \tauò }\mu\tau\varrhoิv\vee\propto[\delta
    \tauò v\tau\varrhoิv\xi\alpha\varepsilon
    \tauò \xi\tau\hat{v o \alpha S}
    \tauò o\tau\hat{v}\pi\alpha\zeta
```

```
5 \tauò \pi\tau\hat{\omega}v\rho\alpha\eta
    \tauò \varphi \tau\hat{\omega}v\rho\alpha,
    \tauò \rho\tau\hat{\omega}v\quad\overline{\alpha}
    ii
    [ -ca.?- ]o
    [-ca.?- ] <\gamma\kappa[-ca.?- ]
    [-ca.?- ] <\gamma\kappa[-ca.?- ]
    [ -ca.?- ]< \gamma [[-ca.?- ]
C
    [\tau\grave{\alpha}\alpha\lambda]
    [\tau\alphà\alpha<\kappa]
    [\tau\alphà \beta & ]
    [\tau\grave{\alpha}\beta<1\varepsilon\xi]
5 [\tau\grave{\alpha}\gamma l]
    [\tau\grave{\alpha}\gamma<l \xi]
    [\tau\grave{\alpha}\delta l \lambda]
    [\tau\grave{\alpha}\delta\angle ı\kappa]
    \tau\alpha}\varepsilon
10[\tau\alphà] }\varepsilon<\zeta
    [\tau\grave{\alpha}]\varsigma\varepsilon
    [\tau]\alphà }\varsigma<\varepsilon
    [\tau\grave{\alpha}]\zeta\varepsilon\lambda
    [\tau\grave{\alpha}]\zeta\angle\delta
15 \tau\alphà \eta \delta [\xi]
    \tau\alpha}\eta<\delta[\lambda
    \tau\alpha}0\delta[\kappa
    \tau\alpha}0<\delta[1\varepsilon
    \tau\alpha}1
20\tau\alphà }1\angle\gamma\xi
    \tau\alpha}1\alpha\gamma
```

$\tau \grave{\alpha} 1 \alpha \angle \gamma \kappa$
$\tau \grave{\alpha} ~ \beta \gamma[1 \varepsilon]$
$\tau \alpha{ }_{\alpha} \beta<\gamma 1 \beta$
${ }^{25} \tau \grave{\alpha} \mathrm{r} \gamma \gamma[1]$
D
$[\tau \grave{\alpha} 1 \gamma<\gamma 1 \xi]$
$\left[\begin{array}{lll}\tau & \kappa & \gamma \\ \imath & \lambda\end{array}\right]$
[ $\tau \grave{\alpha} \tau<\gamma \iota \kappa$ ]
[ $\tau \alpha{ }^{1} \varepsilon \angle$ ]
5 [ $\tau \grave{\alpha} 1 \varepsilon \angle \angle \xi]$
$[\tau \grave{\alpha} 15<\lambda]$
[ $\tau \alpha \dot{\alpha} 15 \angle \angle \kappa$ ]
[ $\tau \grave{\alpha} \iota<\iota \varepsilon$ ]
$[\tau \grave{\alpha} \imath \zeta \angle \angle \varepsilon \xi]$
$10[\tau \alpha ̀ \eta<1 \varepsilon \lambda]$
$[\tau] \grave{\alpha} \rightsquigarrow[\angle \angle \mathrm{l} \xi]$
$\tau \grave{\alpha} 1 \theta[\angle \mathrm{l} \lambda]$
$\tau \grave{\alpha} 1 \theta<\angle \mathrm{l} \kappa$
$\tau \grave{\alpha} \kappa \omega$
$15 \tau \dot{\alpha} \kappa \angle \omega \xi$
$\tau \alpha ̀ \alpha \alpha \omega \lambda$
$\tau \grave{\alpha} \kappa \alpha \angle \omega \kappa$
$[\tau \dot{\alpha}] \kappa \beta \omega \varepsilon \varepsilon$
$[\tau \grave{\alpha}] \kappa \beta \ll \delta$
20 [ $\tau \grave{\alpha}] \kappa \gamma \omega 1$
$[\tau] \alpha ̀ \kappa \gamma \angle \omega \imath \xi$
$[\tau] \alpha$ к $\kappa \omega \imath \lambda$
$[\tau] \grave{\alpha} \kappa \delta \angle[\omega \lambda]$
$[\tau] \underset{\text { 人 }}{ } \kappa \varepsilon[\omega]$. .
$25[\tau] \alpha \dot{\alpha} \kappa\llcorner\omega \leqslant \xi$
$[\tau] \alpha \dot{\alpha} \kappa \varsigma \angle \gamma \lambda$
$[\tau \dot{\alpha}] \kappa \varsigma \angle \angle \gamma[\kappa]$

23 corr. ex $\delta 108$ corr. ex $\sigma \pi 143$ corr. ex $\sigma \kappa 249$ corr. ex 269 l. $\Theta$ (corr) 273 corr. ex
 (corr) $349 \quad l . \backslash \dot{\eta} / \mu(v \rho \iota \alpha ́ \delta \varepsilon \varsigma) \backslash \dot{\mathrm{A}} /$ (corr)
$2 \times 1=2 ; 2 \times 2=4 ; 2 \times 3=6 ; 2 \times 4=8 ; 2 \times 5=10 ; 2 \times 6=12 ; 2 \times 7=14 ;$
$2 \times 8=16 ; 2 \times 9=18 ; 2 \times 10=20 ; 3 \times 1=3 ; 3 \times 2=6 ; 3 \times 3=9 ; 3 \times 4=12 ; 3 \times 5=15 ; 3 \times 6=18 ; 3 \times$ $7=21 ; 3 \times 8=24 ; 3 \times 9=27 ; 3 \times 10=30 ; 4 \times 1=4 ; 4 \times 2=8 ; 4 \times 3=12 ; 4 \times 4=16 ; 4 \times 5=20 ; 4 \times 6$ $=24 ; 4 \times 7=28 ; 4 \times 8=32 ; 4 \times 9=36 ; 4 \times 10=40 ; 5 \times 1=5 ; 5 \times 2=10 ; 5 \times 3=15 ; 5 \times 4=20$;
$5 \times 5=25 ; 5 \times 6=30 ; 5 \times 7=35 ; 5 \times 8=40 ; 5 \times 9=45 ; 5 \times 10=50 ; 6 \times 1=6 ; 6 \times 2=12 ; 6 \times 3=18$; $6 \times 4=24 ; 6 \times 5=30 ; 6 \times 6=36 ; 6 \times 7=42 ; 6 \times 8=48 ; 6 \times 9=54 ; 6 \times 10=60 ; 7 \times 1=7 ; 7 \times 2=14$; $7 \times 3=21 ; 7 \times 4=28 ; 7 \times 5=35 ; 7 \times 6=42 ; 7 \times 7=49 ; 7 \times 8=56 ; 7 \times 9=63 ; 7 \times 10=70 ; 8 \times 1=8$; $8 \times 2=16$;
$8 \times 3=24 ; 8 \times 4=32 ; 8 \times 5=40 ; 8 \times 6=48 ; 8 \times 7=56 ; 8 \times 8=64 ; 8 \times 9=72 ; 8 \times 10=80 ; 9 \times 1=9$; $9 \times 2=18 ; 9 \times 3=27 ; 9 \times 4=36 ; 9 \times 5=45 ; 9 \times 6=54 ; 9 \times 7=63 ; 9 \times 8=72 ; 9 \times 9=81 ; 9 \times 10=90$; $10 \times 1=10 ; 10 \times 2=20 ; 10 \times 3=30 ; 10 \times 4=40 ; 10 \times 5=50 ; 10 \times 6=60 ; 10 \times 7=70 ; 10 \times 8=80$;
$10 \times 9=90 ; 10 \times 10=100 ; 20 \times 1=20 ; 20 \times 2=40 ; 20 \times 3=60 ; 20 \times 4=80 ; 20 \times 5=100 ; 20 \times 6=$ $120 ; 20 \times 7=140 ; 20 \times 8=160 ; 20 \times 9=180 ; 20 \times 10=200 ; 30 \times 1=30 ; 30 \times 2=60 ; 30 \times 3=90 ; 30$ x $4=120 ; 30 \times 5=150 ; 30 \times 6=180 ; 30 \times 7=210 ; 30 \times 8=240 ; 30 \times 9=270 ; 30 \times 10=300 ; 40 \times 1$ = 40; $40 \times 2=80 ; 40 \times 3=120$;
$40 \times 4=160 ; 40 \times 5=200 ; 40 \times 6=240 ; 40 \times 7=280 ; 40 \times 8=320 ; 40 \times 9=360 ; 40 \times 10=400 ; 50$ х $1=50 ; 50 \times 2=100 ; 50 \times 3=150 ; 50 \times 4=200 ; 50 \times 5=250 ; 50 \times 6=300 ; 50 \times 7=350 ; 50 \times 8=$ $400 ; 50 \times 9=450 ; 50 \times 10=500 ; 60 \times 1=60 ; 60 \times 2=120 ; 60 \times 3=180 ; 60 \times 4=240 ; 60 \times 5=300$; $60 \times 6=360 ; 60 \times 7=420 ; 60 \times 8=480 ; 60 \times 9=540 ; 60 \times 10=600$;
$70 \times 1=70 ; 70 \times 2=140 ; 70 \times 3=210 ; 70 \times 4=280 ; 70 \times 5=350 ; 70 \times 6=420 ; 70 \times 7=490 ; 70 \times$ $8=560 ; 70 \times 9=630 ; 70 \times 10=700 ; 80 \times 1=80 ; 80 \times 2=160 ; 80 \times 3=240 ; 80 \times 4=320 ; 80 \times 5=$ $400 ; 80 \times 6=480 ; 80 \times 7=560 ; 80 \times 8=640 ; 80 \times 9=720 ; 80 \times 10=800 ; 90 \times 1=90 ; 90 \times 2=180$; $90 \times 3=270 ; 90 \times 4=360 ; 90 \times 5=450 ; 90 \times 6=540 ; 90 \times 7=630 ; 90 \times 8=720 ; 90 \times 9=810 ; 90 \times$ $10=900 ; 100 \times 1=100$;
$100 \times 2=200 ; 100 \times 3=300 ; 100 \times 4=400 ; 100 \times 5=500 ; 100 \times 6=600 ; 100 \times 7=700 ; 100 \times 8=$ $800 ; 100 \times 9=900 ; 100 \times 10=1,000 ; 200 \times 1=200 ; 200 \times 2=400 ; 200 \times 3=600 ; 200 \times 4=800 ; 200$ x $5=1,000 ; 200 \times 6=1,200 ; 200 \times 7=1,400 ; 200 \times 8=1,600 ; 200 \times 9=1,800 ; 200 \times 10=2,000 ; 300$ х $1=300 ; 300 \times 2=600 ; 300 \times 3=900 ; 300 \times 4=1,200 ; 300 \times 5=1,500 ; 300 \times 6=1,800 ; 300 \times 7=$ 2,$100 ; 300 \times 8=2,400 ; 300 \times 9=2,700 ; 300 \times 10=3,000$;
$400 \times 1=400 ; 400 \times 2=800 ; 400 \times 3=1,200 ; 400 \times 4=1,600 ; 400 \times 5=2,000 ; 400 \times 6=2,400 ; 400$ x $7=2,800 ; 400 \times 8=3,200 ; 400 \times 9=3,600 ; 400 \times 10=4,000 ; 500 \times 1=500 ; 500 \times 2=1,000 ; 500 \times$ $3=1,500 ; 500 \times 4=2,000 ; 500 \times 5=2,500 ; 500 \times 6=3,000 ; 500 \times 7=3,500 ; 500 \times 8=4,000 ; 500 \times 9$ $=4,500 ; 500 \times 10=5,000 ; 600 \times 1=600 ; 600 \times 2=1,200 ; 600 \times 3=1,800 ; 600 \times 4=2,400 ; 600 \times 5=$ 3,$000 ; 600 \times 6=3,600 ; 600 \times 7=4,200 ; 600 \times 8=4,800$;
$600 \times 9=5,400 ; 600 \times 10=6,000 ; 700 \times 1=700 ; 700 \times 2=1,400 ; 700 \times 3=2,100 ; 700 \times 4=2,800$; $700 \times 5=3,500 ; 700 \times 6=4,200 ; 700 \times 7=4,900 ; 700 \times 8=5,600 ; 700 \times 9=6,300 ; 700 \times 10=7,000$;
$800 \times 1=800 ; 800 \times 2=1,600 ; 800 \times 3=2,400 ; 800 \times 4=3,200 ; 800 \times 5=4,000 ; 800 \times 6=4,800$;
$800 \times 7=5,600 ; 800 \times 8=6,400 ; 800 \times 9=7,200 ; 800 \times 10=8,000 ; 900 \times 1=900 ; 900 \times 2=1,800$;
$900 \times 3=2,700 ; 900 \times 4=3,600$;
$900 \times 5=4,500 ; 900 \times 6=5,400 ; 900 \times 7=6,300 ; 900 \times 8=7,200 ; 900 \times 9=8,100 ; 900 \times 10=9,000$; $1,000 \times 1=1,000 ; 1,000 \times 2=2,000 ; 1,000 \times 3=3,000 ; 1,000 \times 4=4,000 ; 1,000 \times 5=5,000 ; 1,000 \times$ $6=6,000 ; 1,000 \times 7=7,000 ; 1,000 \times 8=8,000 ; 1,000 \times 9=9,000 ; 1,000 \times 10=10,000 ; 2,000 \times 1=$ 2,$000 ; 2,000 \times 2=4,000 ; 2,000 \times 3=6,000 ; 2,000 \times 4=8,000 ; 2,000 \times 5=10,000$;
$2,000 \times 6=12,000 ; 2,000 \times 7=14,000 ; 2,000 \times 8=16,000 ; 2,000 \times 9=18,000 ; 2,000 \times 10=20,000 ;$ $3,000 \times 1=3,000 ; 3,000 \times 2=6,000 ; 3,000 \times 3=9,000 ; 3,000 \times 4=12,000 ; 3,000 \times 5=15,000 ; 3,000$ $\mathrm{x} 6=18,000 ; 3,000 \times 7=21,000 ; 3,000 \times 8=24,000 ; 3,000 \times 9=27,000 ; 3,000 \times 10=30,000 ; 4,000 \times$ $1=4,000 ; 4,000 \times 2=8,000 ; 4,000 \times 3=12,000 ; 4,000 \times 4=16,000 ; 4,000 \times 5=20,000$;
$4,000 \times 6=24,000 ; 4,000 \times 7=28,000 ; 4,000 \times 8=32,000 ; 4,000 \times 9=36,000 ; 4,000 \times 10=40,000 ;$ $5,000 \times 1=5,000 ; 5,000 \times 2=10,000 ; 5,000 \times 3=15,000 ; 5,000 \times 4=20,000 ; 5,000 \times 5=25,000 ;$ $5,000 \times 6=30,000 ; 5,000 \times 7=35,000 ; 5,000 \times 8=40,000 ; 5,000 \times 9=45,000 ; 5,000 \times 10=50,000 ;$ $6,000 \times 1=6,000$;
$6,000 \times 2=12,000 ; 6,000 \times 3=18,000 ; 6,000 \times 4=24,000 ; 6,000 \times 5=30,000 ; 6,000 \times 6=36,000 ;$ $6,000 \times 7=42,000 ; 6,000 \times 8=48,000 ; 6,000 \times 9=54,000 ; 6,000 \times 10=60,000 ; 7,000 \times 1=7,000 ;$ $7,000 \times 2=14,000 ; 7,000 \times 3=21,000 ; 7,000 \times 4=28,000 ; 7,000 \times 5=35,000 ; 7,000 \times 6=42,000$; $7,000 \times 7=49,000 ; 7,000 \times 8=56,000$;
$7,000 \times 9=63,000 ; 7,000 \times 10=70,000 ; 8,000 \times 1=8,000 ; 8,000 \times 2=16,000 ; 8,000 \times 3=24,000 ;$ $8,000 \times 4=32,000 ; 8,000 \times 5=40,000 ; 8,000 \times 6=48,000 ; 8,000 \times 7=56,000 ; 8,000 \times 8=64,000$; $8,000 \times 9=72,000 ; 8,000 \times 10=80,000 ; 9,000 \times 1=9,000 ; 9,000 \times 2=18,000 ; 9,000 \times 3=27,000$; $9,000 \times 4=36,000$;
$9,000 \times 5=45,000 ; 9,000 \times 6=54,000 ; 9,000 \times 7=63,000 ; 9,000 \times 8=72,000 ; 9,000 \times 9=81,000$; $9,000 \times 10=90,000 ; 10,000 \times 1=10,000 ; 10,000 \times 2=20,000 ; 10,000 \times 3=30,000 ; 10,000 \times 4=$ 40,$000 ; 10,000 \times 5=50,000 ; 10,000 \times 6=60,000 ; 10,000 \times 7=70,000 ; 10,000 \times 8=80,000 ; 10,000 \times$ $9=90,000 ; 10,000 \times 10=100,000$;
the whole of 2 is 2 ; the $2^{\prime}$ of 3 is $1+2^{\prime}$; the $3^{\prime}$ of 4 is $1+3^{\prime}$; the $4^{\prime}$ of 5 is $1+4^{\prime}$; the $5^{\prime}$ of 6 is $1+5^{\prime}$; the $6^{\prime}$ of 7 is $1+6^{\prime}$; the $7^{\prime}$ of 8 is $1+7^{\prime}$; the $8^{\prime}$ of 9 is $1+8^{\prime}$; the $9^{\prime}$ of 10 is $1+9^{\prime}$; the $9^{\prime} \ldots$ (?); the (?); the $900^{\prime} \ldots$; the $800^{\prime} \ldots$; the $700^{\prime} \ldots$; (?)
the $40^{\prime}$ of 50 is $1+4^{\prime}$; the $50^{\prime}$ of 60 is $1+5^{\prime}$; the $60^{\prime}$ of 70 is $1+6^{\prime}$; the $70^{\prime}$ of 80 is $1+7^{\prime}$; the $80^{\prime}$ of 90 is $1+8^{\prime}$; the $90^{\prime}$ of 100 is $1+9^{\prime}$; the $100^{\prime}$ of (?) is $1+(?)$;

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...2' 3' 20'...; ...2' 3' 20'...; ...2' 3' 10'...
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$1=30^{\prime} ; 11 / 2=20^{\prime} ; 2=15^{\prime} ; 21 / 2=15^{\prime}+60^{\prime} ; 3=10^{\prime} ; 31 / 2=10^{\prime}+60^{\prime} ; 4=10^{\prime}+30^{\prime} ; 41 / 2=10^{\prime}+20^{\prime} ;$ $5=6^{\prime} ; 51 / 2=6^{\prime}+60^{\prime} ; 6=5^{\prime} ; 61 / 2=5^{\prime}+60^{\prime} ; 7=5^{\prime}+30^{\prime} ; 71 / 2=4^{\prime} ; 8=4^{\prime}+60^{\prime} ; 81 / 2=4^{\prime}+30^{\prime} ; 9=$ $4^{\prime}+20^{\prime} ; 91 / 2=4^{\prime}+15^{\prime} ; 10=3^{\prime} ; 101 / 2=3^{\prime}+60^{\prime} ; 11=3^{\prime}+30^{\prime} ; 111 / 2=3^{\prime}+20^{\prime} ; 12=3^{\prime}+15^{\prime} ; 121 / 2$ $=3^{\prime}+12^{\prime}$
$13=3^{\prime}+10^{\prime} ; 131 / 2=3^{\prime}+10^{\prime}+60^{\prime} ; 14=3^{\prime}+10^{\prime}+30^{\prime} ; 141 / 2=3^{\prime}+10^{\prime}+20^{\prime} ; 15=2^{\prime} ; 151 / 2=2^{\prime}+$ $60^{\prime} ; 16=2^{\prime}+30^{\prime} ; 161 / 2=2^{\prime}+20^{\prime} ; 17=2^{\prime}+15^{\prime} ; 171 / 2=2^{\prime}+15^{\prime}+60^{\prime} ; 18=2^{\prime}+15^{\prime}+30^{\prime} ; 181 / 2=2^{\prime}$ $+10^{\prime}+60^{\prime} ; 19=2^{\prime}+10^{\prime}+30^{\prime} ; 191 / 2=2^{\prime}+10^{\prime}+20^{\prime} ; 20=3^{\prime \prime} ; 201 / 2=3^{\prime \prime}+60^{\prime} ; 21=3^{\prime \prime}+30^{\prime} ; 211 / 2=$
$3^{\prime \prime}+20^{\prime} ; 22=3^{\prime \prime}+15^{\prime} ; 221 / 2=2^{\prime}+4^{\prime} ; 23=3^{\prime \prime}+10^{\prime} ; 231 / 2=3^{\prime \prime}+10^{\prime}+60^{\prime} ; 24=3^{\prime \prime}+10^{\prime}+30^{\prime} ; 241 / 2$ $=3^{\prime \prime}+10^{\prime}+20^{\prime} ; 25=3^{\prime \prime}+6^{\prime} ; 251 / 2=3^{\prime \prime}+6^{\prime}+60^{\prime} ; 26=2^{\prime}+3^{\prime}+30^{\prime} ; 261 / 2=2^{\prime}+3^{\prime}+20^{\prime}$
ii. 23 I discern a triangular delta underneath beta; the scribe evidently corrected the mistake by overwriting it.
§21 iii.49 The fibers are almost entirely lost between the traces of theta and the delta written at right. Although this line's product is only lacking a nu (which I have restored in the edition), the lacuna is large enough for two letters. I can only speculate as to the lacuna's contents; the scribe may have made a copying error requiring deletion of a letter, written an unusually large nu, or avoided writing on an area of weak fibers.
iv. 76 Nu and delta are written in combination: the right hasta of the former is also the left diagonal of the latter.
iv. 88 At its base, the left hasta of pi curls backward and up, nearly touching the left edge of the crossbar. The resulting traces resemble an unusually narrow omicron, but it is unlikely that the scribe write that letter in error: the left leg of pi occasionally curls backward in this way (cf. vii.151).
v. 108 The scribe mistakenly wrote pi instead of mu ; as at ii.23, the correction was made by overwriting the original letter.
viii. 186 Immediately below the apex of alpha there is a dot of ink which gives the impression that alpha has a horizontal base. It is more probably stray ink.
§26 viii. 189 Theta is written slightly below the line.
§27 x. 249 The scribe appears to have originally wrote the product of x .249 rather low in the line - almost in between x. 249 and x. 250 . The number is unclear, but it was a two-digit product and therefore certainly incorrect. Correcting the mistake required scribbling it out and writing the correct product in the correct space, which was still vacant thanks to the placement of the original entry.
xi. 269 The scribe made a mistake, entering $\mathrm{A}(=1,000)$ instead of $\Theta(=9,000)$ as the product of 1,000 and 9 . One might alternatively read A as a mu [i.e., $\mu(v \rho ı \alpha ́ c)]$, but that analysis is no more correct mathematically, and fails to account for the traces resembling the hook on a numeral in the thousands.
xi. 270 Here and at xi.275, the scribe indicates explicitly the number of myriads with a superscripted alpha. Subsequently, however, a single myriad is frequently indicated solely by mu: e.g., xii. 294 : $\Delta \delta$ $\mu$ (vpı⿳亠 $\varsigma) \backslash S /(4,000 \times 4=16,000)$; xiv. $322: Z \beta(\mu(v \rho ı \grave{\alpha} \varsigma)) \backslash \Delta /(7,000 \times 2=14,000)$. But the scribe's practice is inconsistent: cf. xvi.351-353.
$\$ 30$ xi. 273 The scribe appears to have originally wrote epsilon. Correction was made by overwriting the incorrect numeral.
xii.277-278 Fragment 4 has been placed here by virtue of the reading: $\mu$ (vрíac) $S(=16,000)$. The only other places in the table that number should appear are xii. 294 or xv.332, where it is already preserved and read with certainty.
xiii. 308 There is sufficient blank papyrus to the right of eta for the product to be entered; it is unclear whether the ink has been abraded or whether the scribe erred in omitting it.
$\$ 33$ xiii. 311 Both the paragraphos and the traces below it are difficult to discern. But the line should include the first calculation for $6,000: S \propto S$ (i.e., $6,000 \times 1=6,000$ ).
§34 xiv.312-313 Two observations support the placement of fragment 5 in this position: it appears to preserve the top of the column; and its second line's reading - $\mu\left(v \rho \rho^{\prime} \alpha \varsigma\right) H(=18,000)$ - is expected by the reconstruction of column xiv. Other possible placements for the numeral 18,000 are less likely: if it were xii. 279 it would overlap physically with the small fragment already placed at xii.277-278; if it were xii. 286 it would require reading $\mu$ (vคíac) $\mathrm{E}(=15,000)$ in the line above, which does not suit the traces; if it were xv. 342 it would require reading $\Theta(=9,000)$ in the line above, which is similarly unsuitable for the traces. Although I admit that the reading $\mu\left(v \rho^{\prime} \alpha \varsigma\right) B(=12,000)$ in xiv. 312 is difficult, it is better than the alternatives.
$\$ 35$ xiv. 323 Only two flecks of ink are visible, which I both presume are the digits written above $\mu(v \rho 1 \alpha ́ \delta \varepsilon \varsigma)$ and restore as required by the calculation.
§36 xv. 336 There are two curiosities in this line. The first is that the number of myriads was initially entered incorrectly (seemingly as stigma) before the correct form (delta) was overwritten. The second is that the product's second digit is also wrong, but was in this case left uncorrected. The fragment preserves enough of this and the subsequent column to both guarantee its position in the table and understand the mistake.
§37 xv.338-340 The correct analysis of the initial etas in the first column of fragment 6 - the first of which is level with xiv. 322 - is facilitated by the reconstruction of the columns. With allowances for irregular spacing of the lines across columns, an eta opposite the first two entries for zeta is almost certainly one of the last three entries for that multiplicand. It seems reasonable to conclude that column xv therefore had seventeen lines, while columns xiv and xvi both had sixteen.
§38 xvi.347-349 The readings are not in doubt, but point to faulty arithmetic in the lines immediately before xvi. $350(9,000 \times 10=90,000)$. Evidently, in each of the three preceding entries, the number of myriads has been omitted (despite being greater than one), and the digit for thousands is incorrect (as it belongs rather in the entry above). In other words, a product was dropped out somewhere before xvi. 350 .
§39 A. 1 The formula according to which the first nine lines of this fragment are restored (i.e., ) is based on fragment B's first column, whose formula is .
$\$ 40$ A. 10-14 Infrared images render the text of the ed.pr. untenable. The preceding lines’ sequence suggests that the calculation should involve a tenth of another quantity, but iota is not easily read: the traces better resemble a theta. Below the paragraphos, we appear to be in an entirely different section of the table, one organized in descending order. The first fraction (A.11) is illegible, but below it I read sampi, omega, and psi with tick marks $\left(=900^{\prime}, 800^{\prime}, 700^{\prime}\right)$. Line 11 , in other words, appears to begin a new sequence involving fractions of hundred parts. But unlike elsewhere in the table, these fractions are presented in descending order. If the text is the same one as elsewhere on fragments $A$ and $B$, we might restore, e.g., tò $\lambda^{\prime}\left[\tau \hat{\uparrow} \mu\left(v \rho^{\prime} \alpha \varsigma\right) \alpha \theta^{\prime}\right]$ (A.12).
$\$ 41$ B i. 7 The paragraphos below this line indicates it is the final entry in a section, but several of the values are read with difficulty, making the restoration uncertain. The first, a fraction with a tick, consists of an upright stroke with very faint traces on either side at its top. The pattern of the preceding lines suggests rho, but iota or even koppa cannot be definitively excluded. Only two flecks of ink from the second value survive: one at letter-bottom and one on the upper-right side. The quotient consist of two digits: the traces of the first are the apex of an angular letter - almost certainly alpha - surmounted by an overstroke; of the second only lunate traces at right survive.
§42 Below the paragraphos there are faint traces of one line in the same hand - apparently a prior version of the text that has been washed out - as well as a second text in a much larger hand, which appears to have been erased with much less success. On the infrared image, the ink appears to have been smeared rather than washed away. The layout of the second column suggests that the area below the paragraphos was the lower margin.
§43 C. 11 Following epsilon, there are additional traces of ink - two faint hastas that recline slightly to the left (cf. the usual shape of iota, which inclines slightly to the right). But the entry is complete without the need for anything more.
§44 C. 25 A bottom margin measuring 1.9 cm follows this line, and to the left of the final six lines there also appears 4.2 cm of blank space, evidently a margin. That layout permits several inferences regarding the structure of both the column and this particular section of the text: 1) The table on this sheet began at the top of the column, and its first eight lines have been lost. Based on the size and spacing of the letters, the sheet would have measured at least 17 cm , not including an upper margin and a table header; 2) The second column is preserved on fragment $D$, which is missing nine lines from its top; 3) The third column, were it extant, would have contained seven lines.
D. 24 A difficult reading. The diagonal tick following the fraction is clear but there are no traces of the fraction. The traces that follow are consistent with stigma, which permits the restoration of the calculation. But there are traces of another hasta following stigma, which I cannot explain.

## Appendices

Appendix A: Dimensions and Contents of the Fragments

| Fr. 1 | col. ii-xiii | $21 \times 42 \mathrm{~cm}$ |
| :--- | :--- | :--- |
| Fr. 2 | ix.202-206, x.229-239, xi.256-258 | $8.1 \times 8.5 \mathrm{~cm}$ |
| Fr. 3 | xii.282-283 | $2.2 \times 2.9 \mathrm{~cm}$ |
| Fr. 4 | xii.277-278 | $1.9 \times 1.8 \mathrm{~cm}$ |
| Fr. 5 | xiv.312-313 | $2.3 \times 2.5 \mathrm{~cm}$ |
| Fr. 6 | xiv.321-322, xv.338-340 | $5 \times 8 \mathrm{~cm}$ |
| Fr. 7 | xv.331-336, xvi.347-354 | $8.7 \times 10.5 \mathrm{~cm}$ |
| Fr. A |  | $8.9 \times 4.8 \mathrm{~cm}$ |
| Fr. B |  | $6.9 \times 9.5 \mathrm{~cm}$ |
| Fr. C |  | $12 \times 9 \mathrm{~cm}$ |
| Fr. D |  | $11.9 \times 5.5 \mathrm{~cm}$ |

Appendix B: Record of Objects from House 125 / B163 / C163

| $\mathbf{2 4 - 1 2 5 C}$ |  |
| :--- | :--- |
| A | Wooden spindle whirl. |
| B | Glass. |
| 28-B163A |  |
| A | Papyrus. (P.Mich. inv. 5375-5378) |
| B | Shell amulet on fragment of braided leather. |
| C | End of bronze necklace with one glass bead. |
| D | Beads. |
| E | Bronze frag. |
| F | 7 coins. |
| G | Segmental glass inlay, with two small bored hollows on one edge. |
| 28-B163A* |  |


| A | Stirring stick. |
| :---: | :---: |
| B | Wooden scythe handle (?). |
| C | Strap handle, perhaps door bar catch, wood. |
| D | Part of door panel, roughly hewn. |
| E | 3 tiny wooden spindle whorls. |
| F | Tiny wooden hammer head. |
| G | Colorless glass fragments to house; many green bowl frag., and iridescent pieces NTH. |
| G | Colorless glass fragments to house; many green bowl frag., and iridescent pieces NTH. |
| H | 2 head rings of palm fibre. |
| J | Rope muzzle. |
| K | Terracotta votive arm with hole for suspension. |
| L | Frag. gourd husk. |
| M | Blue glaze frag. |
| N | Papyrus ca. m. 2.00 below base of east wall. (¢ P.Mich. inv. 5379) |
| O | Demotic Papyrus. (¢P P.Mich. inv. 5380) |
| P | Beads. |
| Q | Plaster of mud with traces of painting, a yellow panel with black border, with decoration in white and red dots. |
| R | Very tiny blue beads on string, 18 beads. |
| S | Coin |
| T | Bronze frag. |
| a | Vessel: 62 (VII) NTH. |
| b | Vessel: 5-55 (XI) two only to house. |
| c | Vessel: 217 (XI) NTH. |
| d | Vessel: 2-159 (VII, $\mathrm{b}^{48}$ ) one only to house. |
| e | Vessel: Frag. of shallow bowl, flat base, good glaze, perhaps Samian. |
| 29-B163* |  |
| A | Blue glaze frags. |
| B | Wooden comb. |
| C | Pot sherd with decoration. |
| D | 11 mud loom weights NTH. |
| E | Coarsely plaited palm leaf basket, oval shaped. |
| F | Section of a rounded bone pierced bead. |
| G | Glass inlay set. |
| H | Beads. |
| J | Papyrus. (¢P P.Mich. inv. 5663) |
| K | Frags. of terracotta Harpocrates. |
| L | 17 coins. |
| M | Flat piece of bone, pointed end. |
| N | Vessel: Pottery lamp, type VIII. |
| O | Frag. of bronze band ring. |
| P | Vessel: Pottery lamp, type VIII. |
| Q | Wooden comb. |
| R | Ostracon. (CO.Mich. inv. 9307) |
| S | Vessel: Pottery lamp, type III, b. |
| T | Bone die. |

[^8]| U | Terracotta head projections, above head is pierced as if for use as amulet. |
| :--- | :--- |
| V | Wooden wall lamp bracket. |
| W | Vessel: Top of lamp, type V, e, NTH. |
| X | Spindle whorl. |
| Y | Large coarsely plaited palm leaf basket, closely woven. |
| Z | Frag. of sieve NTH. |
| AI | Frag. of large coiled basket, round, NTH. |
| BI | Head ring of palm leaf. |
| CI | A flat bone marker for pots. m. 0.01 long. |
| DI | Ostracon. (C O.Mich. inv. 9308) |
| EI | Small blue glaze amulet. |
| FI | Three beads on a frag of string. |
| GI | Bone marker for pots m. 0.095 long. |
| HI | Small rope hobble |
| JI | Two frags. of yellowish brown glass. Decorated with white streamers and red blob. |
| KI | Frag. of very roughly made terracotta horse. |
| LI | Straight toggle NTH. |
| MI | Rope hobble NTH. |
| NI | Palm fiber sandal NTH. |
| OI | Small round, burnt pottery bead. |
| PI | Frags. of mud plaster with colored decoration. |
| QI | Small turned wooden knob handle of box lid. |
| RI | Small woven grass basket, perhaps a covering for a small pot or glass. |
| SI | Ostracon high in filling. (O.Mich. inv. 9309) |
| TI | Small blue glaze "Bes amulet". |
| a | Vessel: 217, XI, c, i NTH. |
| b | Vessel: 40, XI, a, i |
| c | Vessel: 380, XV, a |
| d | Vessel: V? |
| e | Vessel: 199, type XXXVI, b |
| f | Vessel: XXI, b. |

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Sampson, C. Michael<br>GND: © https://d-nb.info/gnd/104985960X<br>ORCID: © https://orcid.org/0000-0002-2438-7102<br>University of Manitoba<br>mike.sampson@umanitoba.ca


[^0]:    1 Sijpesteijn counted 26 individual fragments; this edition will refer rather to the reassembled pieces: see Appendix A for a breakdown of the the fragments' dimensions and contents.
    $2 \measuredangle$ CTruins (1981: 297) tacitly corrected errors of arithmetic in the ed. pr. at lines 281 (= fr. C.14, cf. my reading), 283 (= fr. C.16, cf. my reading), and 298 ( $=$ fr. D.22), and $\checkmark$ Sesiano (2020: 103) noted errors at lines 92 ( $=$ vi.126), 114 ( $=$ viii.188), 115 (= viii.189), and 242 (=xv.337). But the latter himself errs in criticizing the ed. pr. at lines 227 (= xii.293) and 228 (= xii.294), where he has misinterpreted the papyrus.
    3 The omissions were noted independently by © Sesiano (2020: 103).
    4 Except for inv. 5663a, which is glazed, the fragment groups of inv. 5663 are stored in individually labelled packets distributed across five separate conservation folders.
    5 Field numbers in the Michigan record of objects can be parsed as follows: 29 refers to the excavation season ( $=1929 / 30$ ); B refers to the occupation layer (from top to bottom, A-F, unless unassigned); 163 identifies the structure; * indicates a specific part of the structure, such as that it was found below ground level; $\mathbf{J}$ is an ordinal ( $\mathrm{A}-\mathrm{Z}$ ) distinguishing the objects (or groups of objects) discovered in a room or division within a structure.
    6 Boak (1926: 20).

[^1]:    

[^2]:    8 On the history of digging Kôm Aushim, see, e.g., © Petrie (1891:32); David G. Hogarth to Emily Paterson (General Secretary of the Egypt Exploration Fund), January 11, 1896 (EES Inv. III k 134), cited in © Montserrat (1996: 150); © Grenfell, Hunt, and Hogarth (1900: 19, 21); and © Goodspeed (1898: 347).
    9 When excavations began, this part of Karanis was dubbed "Area B." See 飞 Boak \& Peterson (1931:3-4, 39-40) for a discussion of the excavations' progress.
    $0 \quad \square$ Peterson (1973: 633).
    11 An annotation in the excavation's record of objects reports that house 163 is "B period for 125 ", i.e., the same structure whose earlier occupation level was relabelled.
    12 CTeterson (1973: 576).

[^3]:    3 ① Boak and Peterson (1931:39).

    [ $\checkmark$ Peterson (1973: 381).
    16 The ongoing extraction of sebakh was an ongoing concern for the excavation, especially in the early seasons: see, for example, Boak (1933: 54-55). Regardless of the sebakh-diggers' activities, the Michigan team did not backfill the structures they exposed, meaning that the forces of erosion would have continued undeterred, further imperilling the integrity of the archaeological record in the interim.
    17 For similar cases of retroactive distinction between occupation levels, see [Ⓒlaytor and Verhoogt (2018: 9) on C123 / B202; $\checkmark$ Stephan and Verhoogt (2005) on C167 / B167.
    8 Cf. $\mathbb{C}$ Landvatter (2016: 1497-1502) on a similarly problematic case at Karanis: field number B224B*-M.
    C Peterson (1973: 361).

[^4]:    20 Peterson (1973: 381).
    21 Nicholas Hudson, my collaborator on The Books of Karanis project, is preparing to undertake precisely such an analysis of select ceramic assemblages.
    22 See the introduction to P.Mich. 15686 [ $=\square$ Sijpestein (1982: 2)].
    23 The washed-out text is particularly visible at the bottom of the fifth and sixth columns.

[^5]:    24 In the nomenclature of $\mathbb{C}$ Sesiano (2020: 101), it is an "intermediate table": the papyrus is no. 19 in his catalogue (2020: 102).
    25 The first column is lacunose, so although it is possible the table included the banal calculations for 1, I concur with $\mathbb{C}$ Sesiano (2020: 94-103, esp. 97), who considers it unlikely: see, e.g., his Appendix VII.2, (ऽ 2020: 123-124).
    The scribe is inconsistent when it comes to myriads: see commentary on xi. 270.
    Cf. © TM 64891, © © PSI 8958.
    E.g., (fr. B i.2); (fr. B i.3); (fr. B i.4); etc. The calculation is reminiscent of a problem in Egyptian mathematics, preserved in several forms on 厄 TM 55977: if a fraction is added to one (i.e., ), what fraction of that sum must be subtracted from it to yield one again? The answer is of : if one adds a third to one, for example (i.e., ), one must subtract a quarter of that sum (i.e., ) from it to leave a remainder
    
    29 The denominator, in other words, is one unit less than the numerator. The first few lines of fragment A therefore calculate as follows: $; ;$; etc. The table of division, in other words, analyzes superparticular or epimoric ( $=\dot{\varepsilon} \pi 1 \mu$ ó $\rho \imath \varsigma)$ ratios: see Nicomachus, Introduction to Arithmetic 1.17-23. Epimoric proportions apply in ancient music theory, but they are of practical value in the calculation of interest: see, e.g., $冖$ Burkert (1962: 415).

[^6]:    30 These fragments probably belong to a new sheet. See comm. on fragment C.25.
    31 So, for example, (fr. D.14); (fr. D.15); (fr. D.16), etc. See, ® Bruins (1981: 295) for discussion.

[^7]:     uer 3, nn. 5, 7; ■ Azzarello (2008: 159n1); 『 P.Bastianini 10 p. 77; and © P.Capasso 3, p. 24. See © Azzarello (2018: 95-97) for a concise introduction to tables of division; and $\square$ Sesiano (2020) on historical tables of multiplication.
    33 I know of no parallel for the second text beyond the one mentioned in n. 28; the only other example of a table of thirtieths is
     catalogue in $\begin{gathered} \\ \text { Fowler (1999: 238) indicates that, for tables of division with divisors larger than 19, only one or (at most) two examples }\end{gathered}$ survive.
    34 From the third century CE, the headers of tables of division included a calculation of 6000/x: see Azzarello (2018).

[^8]:    48 The excavation's pottery key includes a correction for this item: it should be IX (narrow mouth jars with flat base) d (long narrow base, tall neck)

