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A Pyramidal Frustum Volume Algorithm in P.Math. Leaf C Recto

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§1 Mathematical problem c1 in P.Math. (Oxyrhynchos? 350–375; TM 92734) was interpreted in the ed.pr. as being concerned with finding the volume of a quadrangular prism, an interpretation that accounted well for the preserved computations.[1] The statement of the problem, however, was far from clear. Here is how it appears in the edition:

ξύλων νέον πηχ̣[ῶν κ̅η̅ -ca.?-]
.̣.̣[.̣.̣].̣ρι.̣οι δακτύλων ιϛ, τ̣ὸ δὲ.̣[-ca.?-]
[δα]κτύλων ι̅β̅, πάχος μὲν α̣π̣ο̣ρ̣ι̣ δακτ̣ύ[λων]
[η̅.̣.̣.̣.̣.̣]φύλλων δακτύλων ϛ̅. εὑρ[εῖν]
[6-7] τ̣ὸ ξύλον.

A fresh beam[?], [28] cubits ...
... 16 fingers, and the ...
12 fingers, thickness ... [8] fingers
... leaves[?] 6 fingers. To find
the beam[?].

§2 The editors were puzzled by the reference to ‘leaves’ in line 4 (φύλλων) and the incomprehensible α̣π̣ο̣ρ̣ι̣ in line 3. The latter characters in fact are very clearly visible, while the traces in line 2 printed in the edition as.̣[.̣.̣].̣ρι.̣οι suggest that something similar was written there, too (Fig. 1). The appearance of the sequence αποροι along with φύλλων in the description of a piece of timber makes it virtually certain that it conceals a reference to the lower part of a tree, from the root(s), ἀπὸ ῥίζης or ἀπὸ ῥιζῶν, as opposed to its upper part, from the foliage, ἀπὸ φύλλων. The terminology comes up in a passage of Theophrastus in which he explains how wood should be chosen for sockets and pivots (*HP* 5.3.7): Ποιοῦσι δὲ αὐτοὺς ἔμπαλιν τιθέντες τὰ ξύλα τό τε ἀπὸ τῆς ῥίζης καὶ τὸ ἀπὸ τοῦ φύλλου. καλοῦσι δὲ οἱ τέκτονες τὸ ἀπὸ τοῦ φύλλου τὸ ἄνω, “They make them (sc. hinges) by reversing the wood ‘from the root’ and ‘from the foliage’—by ‘wood from the foliage’ they mean the upper part wood.”

1 I am grateful to Alexander Jones for commenting on a draft of this paper and to both editors of P.Math. for kindly sharing the image of the relevant codex page with me. I also appreciate the permission to reproduce leaf C recto of the codex granted by the owner of the 'Archimedes Palimpsest'. The diagrams in Fig. 2 and 3 are drawn by Jannis Schramm.

Fig. 1. P.Math. folio C (recto).

§3 That the terminology in the statement of the problem c1 indeed refers to the 'lower' and 'upper' ends of the original tree is further supported by the measurements of the timber because the dimensions of the cross-section 'from the root' (16 by 8 fingers) are larger than those 'from the foliage' (12 by 6 fingers).

References to the lower and upper ends of the timber in the statement of the problem, however, entail a reconsideration of the shape of the object and of the algorithm used for computing its volume. What is described in the text is not a quadrangular prism, but a truncated pyramid with a rectangular base 18 by 8 fingers, a rectangular top 12 by 6 fingers, and a ‘length’, which is presumably its height, of 28 cubits. It can be illustrated with the following diagram (Fig. 2), in which no attempt is made to render the height commensurate with the dimensions of the base and of the top (one should imagine the figure in the diagram stretched out along its vertical dimension so that its height is about 40 times its width at the base):

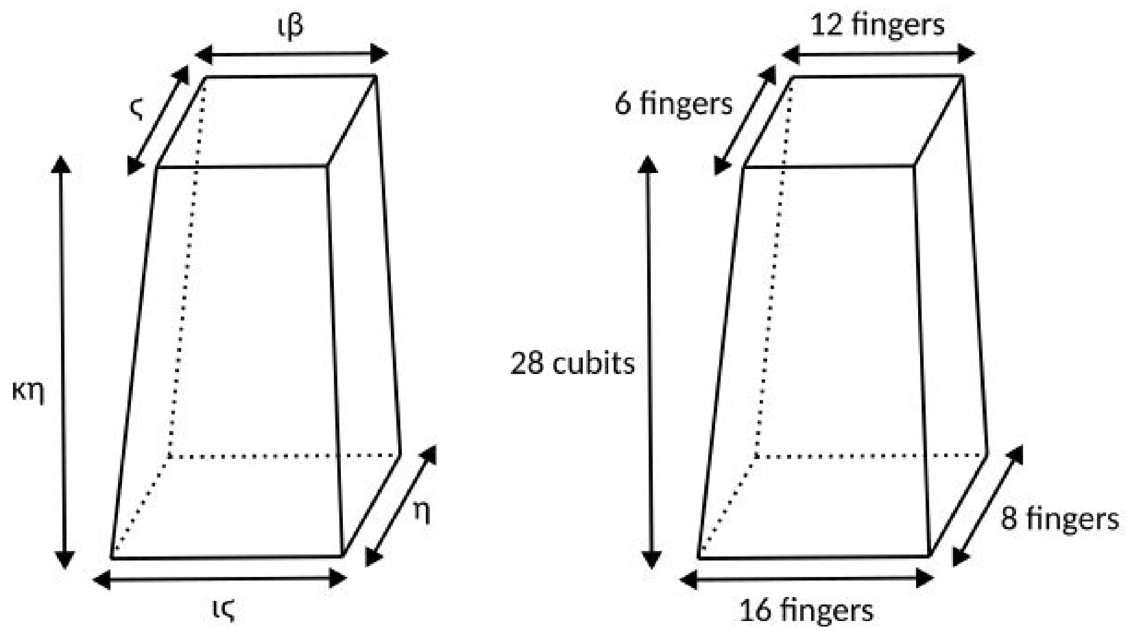


Fig. 2. Reconstruction, not to scale, of the object in problem c1.

§4 The volume of this truncated pyramid is computed as if it were a rectangular prism of the given height (h) with the sides of its bottom face equaling the average of the corresponding sides of the base and of the top: $V = \frac{(a_1 + a_2)}{2} \times \frac{(b_1 + b_2)}{2} \times h$

§5 The algorithm, which produces an approximation and underestimates the volume of the object, is used in pseudo-Heron, *Stereometrica* 25 (Heiberg) to determine the volume of a σφηνίσκος, which has the shape of a truncated pyramid. The approximation formula might be of ancient origin,[2] but P.Math. seems to be its first attestation in the Greek papyrological evidence, and thus we add it to Bagnall and Jones’ collection of algorithms attested in Greek papyri.

§6 **S6C. Pyramidal frustum volume algorithm C (approximate)**. Given a pyramidal frustum having rectangular base with sides a_1 and a_2, rectangular top with sides b_1 and b_2, and height h, find the volume V:

(i) Add a_1 and a_2.
(ii) Divide the sum by 2.
(iii) Add b_1 and b_2.
(iv) Divide the sum by 2.

2 For approximation formulae used to compute the volume of a truncated pyramid and a conic frustum, see Vogel 1930. Note that an algorithm similar to the one for the volume of a truncated pyramid is used in P.Math. problem n1 for the volume of a conic frustum, which is computed as if it were a cylinder with the diameter equaling the average of the lower and upper diameters (S3A), cf. Bagnall and Jones 2019: 42-43.

(v) Multiply the quotient by the quotient obtained in (ii).

(vi) V is the product multiplied by h

$$V = \frac{(a_1 + a_2)}{2} \times \frac{(b_1 + b_2)}{2} \times h$$

Used in: P.Math. c1.

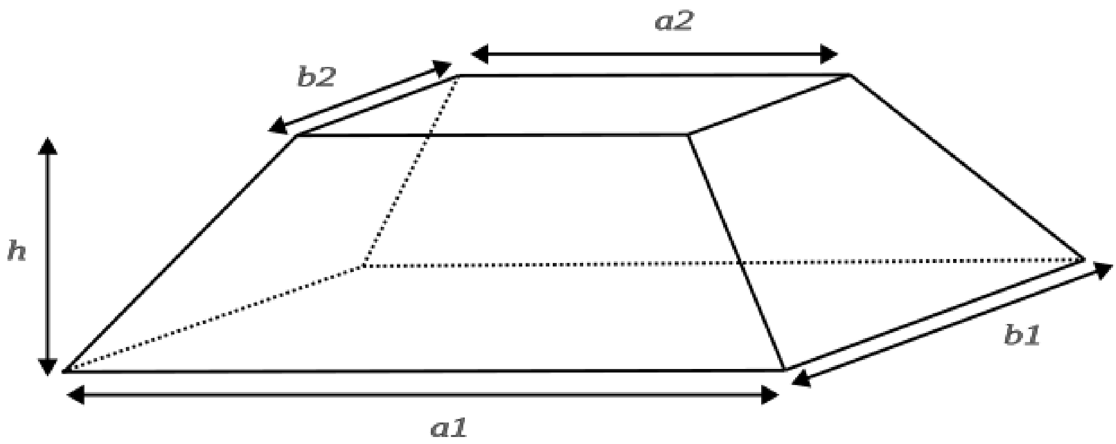


Fig. 3. Diagram for algorithm S6C.

§7 The computations of the volume in the solution of the problem c1 are followed by the conversion of the product—the interim result—to volumetric cubits and fingers. The conversion factors 288 and 12 indicate that the unnamed units in which the final result is expressed are the 3-palm cubit and its fingers.[3]

§8 There remains a question of the description of the wood as ξύλον νέον in the statement of the problem. The adjective might simply mean ‘young’, designating timber that came from a younger as opposed to an older tree. But it is tempting to see in νέον a misspelling of νεῖον, which Aelius Herodianus defines as τῆς νηὸς τὸ πυθμενοειδὲς ξύλον, ‘the keel wood of the ship’ (*De prosod. catholica*, 3,1, p. 356). Hesychius and Photius use the word in the plural and explain it more vaguely as ship-timber (Hesych., s.v. νεῖα· ξύλα, τὰ εἰς κατασκευὴν νεῶν ἐπιτήδεια, ‘wood suitable for building ships’; Photius, s.v. νεῖα: τὰ εἰς κατασκευὴν νεῶν ξύλα, ‘wood for building ships’). The change ει > ε is relatively common, especially before the back vowel ‘o’, and occurs elsewhere in P.Math.[4] If this is the word meant, the object in the problem is ship-timber, and since it is rectangular in cross-section, perhaps the keel or floor of the hull was envisaged rather than a mast.

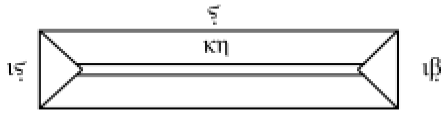
C recto, lines 1–13 (Problem c1)

ξύλων νέον πηχ̣[ῶν κ̅η̅, πλάτος μὲν]
ἀ̣[π]ὸ̣ ῥ̣ίγοι δακτύλων ιϛ, τ̣ὸ δὲ φ̣[ύλλων]
[δα]κτύλων ι̅β̅, πάχος μὲν ῥ̣ι<ζ- > δακτ̣ύ[λων]
[η̅, τὸ δὲ] φύλλων δακτύλων ϛ̅. εὑρ[εῖν]
[-6-7-] τ̣ὸ ξύλον. οὕτω ποιοῦμαι. συντί[θω]
[τὸ πλ]άτος, ι̅ϛ̅ καὶ ι̅β̅· γί(νεται) κ̅η̅· ὧν ἥμισυ [ι̅δ̅. συν]-
[τί]θ̣ο̣μ̣εν τὸ πάχος, {ι}ϛ̅ καὶ η̅· γί(νεται) ι̅δ̅· ὧν ἥμι[συ ζ̅·]

3 For volume cubits of different sizes used for expressing volume of wood and for computation of volume, cf. Lougovaya 2022.

4 Cf. Gignac 1976:257, Bagnall and Jones (2019):12.

[ἐ]π̣ὶ δὲ τ[ὸ]ν ι̅δ̅· γί(νεται) ϙ̅η̅· ἐπὶ τὸ μ̣ῆ̣κος̣, πηχ̣ῶν κ̅η̅·
[γί(νεται)] [Β̅]ψ̣̅μ̣̅δ̣̅. τ̣α̣ῦτα μερίζομαι π̣[α]ρ̣[ὰ] τ̣ὸν σ̅π̅η̣̅·
κα̣ὶ τὰ λυπ̣ὰ̣ ἰς δακτύλους ι̅β̅· γί(νεται) θ̅ καὶ δακτύλ[ων]
ι̅β̅ ϖ̅. οὕτως ἔχει ὁμοίως.



1 *l.* ξύλον **2** *or* ῥιτ̣οι *l.* ῥιζ- **3** corr. ex γακτῦ̣[λων] (?) **7** corr. ex τβ̅ corr. ex κ **10** *l.* λοιπὰ εἰς {ι̅β̅} ed.pr.

'A piece of ship (?) timber, 28 cubits; width from the roots 16 fingers, from the foliage 12 fingers; thickness from the roots 8 fingers, from the foliage 6 fingers. Find [how big?] is the timber. I do it this way: I add the width, 16 and 12, the result is 28, half of which is 14. I add the thickness, 6 and 8, the result is 14, half of which is 7; (multiply) by 14, the result is 98; (multiply) by the length of 28 cubits, the result is 2,744. This I divide by 288 and the remainder into fingers, (divide by) 12, the result is 9 and 12 2/3 fingers. So it holds for similar cases. *Diagram.*'

§10 2 ῥίγ̣οι: it is difficult to decide from the image whether the letter after the first iota is gamma or tau, as its horizontal protrudes slightly to the left. It is certainly not a zeta, cf. also 3n.

§11 3 ἀπὸ ῥι<ζ-> δακτύ̣[λων]: the delta of δακτύ̣[λων] is corrected from what seems to have been a gamma; perhaps the scribe started writing ῥιγ-, as in line 2, but then corrected the gamma to the delta of the subsequent word. There seems to be no sign of an abbreviation, and the phonetic process that could have produced the interchange of γ (or τ) with ζ is not immediately clear, yet it is hardly possible that a different word but ῥίζη, in whatever form, was meant here and in line 2.

§12 4–5 One would expect εὑρεῖν at the end of line 4 to be followed by τὸ στερεὸν τοῦ ξύλου, cf. the formulation of the task in another volume problem, g3 (G verso, lines 11–12), εὑρεῖν | [τὸ στ]ερεόν, 'find the volume', or in similar problems in Didymus, *Mensurae marmorum ac lignorum* 40 and 41 (Heiberg), εὑρεῖν αὐτοῦ τὸ στερεόν, 'to find its volume'. That τὸ ξύλον stands in the nominative and not the genitive, would not be a problem in P.Math., which has multiple instances of the wrong usage of cases,[5] but τὸ στερεόν is certainly too long for the lacuna. Two possibilities suggest themselves: The word for volume may have been written without the article, thus εὑρεῖν | [στερεὸν] τὸ ξύλον, l. εὑρεῖν | [στερεὸν] τοῦ ξύλου. Or, a different expression was used for the question about the size of the piece, such as [πόσων] τὸ ξύλον, 'how big is the wood?' where πόσων would imply πόσων πηχῶν στερεῶν ἐστι τὸ ξύλον, 'of how many volume cubits is the wood?' Since volume units are not named in problems of this type in P.Math. (that is, in a3, b5, c1 and g4), it is likely that πόσων would not be followed by the word designating units either.

§13 7 {ι̅}ϛ̅: I follow the editors in printing and excising the iota, although the squiggle to the left of the stigma does not really look like an iota. It could be that the writer initially miscopied from his original (cf. the omicron in τό earlier in the line, which clearly was first written as a beta with a numeral

5 Bagnall and Jones 2019: 14 provide a useful list of wrongly used cases and other syntactical flaws in the codex.

overstroke). For example, he might have begun to write kappa before switching to stigma, but without caring to delete the left vertical of the kappa. Whatever its genesis, this stroke ought to be excised.

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