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Between Ahmes and Alcuin: P.Bodl. 1 7 Revisited

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§1 Papyrus MS Gr. class. c 96 was edited by R.P. Salomons as P.Bodl. 1 7 under the heading of an ‘arithmetical problem’ (unknown prov., 5th–7th c.; TM 64957).[1] Salomons deciphered most of the text but was unable to read some parts and could find no close parallels to it. Examination of the papyrus has allowed us to make improvements on his edition and to identify similarities with problems attested in the medieval tradition, the earliest examples of which in the Latin West are preserved in Alcuin’s collection of c. AD 800. Some features of the problem’s presentation and solution find forerunners also in the so-called *ꜥḥꜥ* (or *aha*) problems, the majority of which are transmitted in pRhind, a mathematical papyrus copied by a scribe named Ahmes sometime in the middle of the 16th c. BC.

§2 Before discussing the place of P.Bodl. 1 7 within the larger tradition of mathematical problems, we first offer a revised text with translation and commentary.

Reedition of P.Bod. 1 7

§3 The papyrus measures 30.5 cm in width and 13 cm in height and appears to be complete. Its format is unusual and finds parallels in horizontally-oriented inked wooden tablets containing school texts. These tend to have a height 2 to 2.5 times smaller than their width, which typically ranges from c. 25 to 35 cm.[2] The papyrus shows traces of folds and is damaged along a central vertical crease. The fibers are dark and the writing, which runs perpendicular to them, is awkward – Salomons called it ‘utterly unskilled’. Except epsilon, which is often ligatured with what follows, most letters are unligatured and vary in size and ductus. Such a hand is difficult to date, but similarities can be seen in PSI 1 25 (Hermopolis; AD 465); in ‘hand 4’ of P.Flor. 3 287 (Aphrodito; AD 535); and in P.Oxy. 61 4132 (AD 619). These texts span the mid-fifth to first quarter of the seventh century, a date range in keeping with what Salomons proposed.

§4 The beginning of the papyrus is marked by a cross and there is a long, slightly ascending stroke in line 8 after the numeral ρ, which looks similar to a paragraphos and apparently signifies a transition in the text. Ordinal numerals have overstrokes (τὸ $\overline{\alpha}$ in ll. 7 and 9, τὸ $\overline{\beta}$ in l. 9) and the sign for a quarter, which looks like a Latin *d*, has a horizontal line that transects the vertical bar in all but one case (l. 5).

1 We are grateful to staff at the Bodleian Libraries for permission to study Papyrus MS Gr. class. c 96 when we were in Oxford in May 2022. We also thank Alexander Jones and Demokritos Kaltsas for helpful comments on an early draft.

2 Examples include the wooden codex BL Add. MS 37533: 27 × 9.5 cm (unknown prov., 3rd c.; TM 64097 = Cribiore no. 385); a wooden codex in the Ashmolean Museum, T.Bodl.Gk. Inscr. 3019: 23.8 × 11 cm (unknown prov., late 3rd c.; TM 61276 = Cribiore no. 388); another tablet from the Ashmolean, Bodl.Gr. Inscr. 3017: 36.5 × 13.5 cm (unknown prov., 2nd–3rd c.; TM 60718 = Cribiore no. 333); two tablets from the Yale collection, PCtYBR 3678: 27.3 × 13.8 cm (Oxyrhynchus, 470; = TM 61399) and PCtYBR inv. 3675 = P.Yale 4 186–187: 33.6 × 12.5 cm (possibly Oxyrhynchite, 7th c.; TM 975003 and 975004). For the format of school tablets, see also Lougovaya 2023: 223–226.

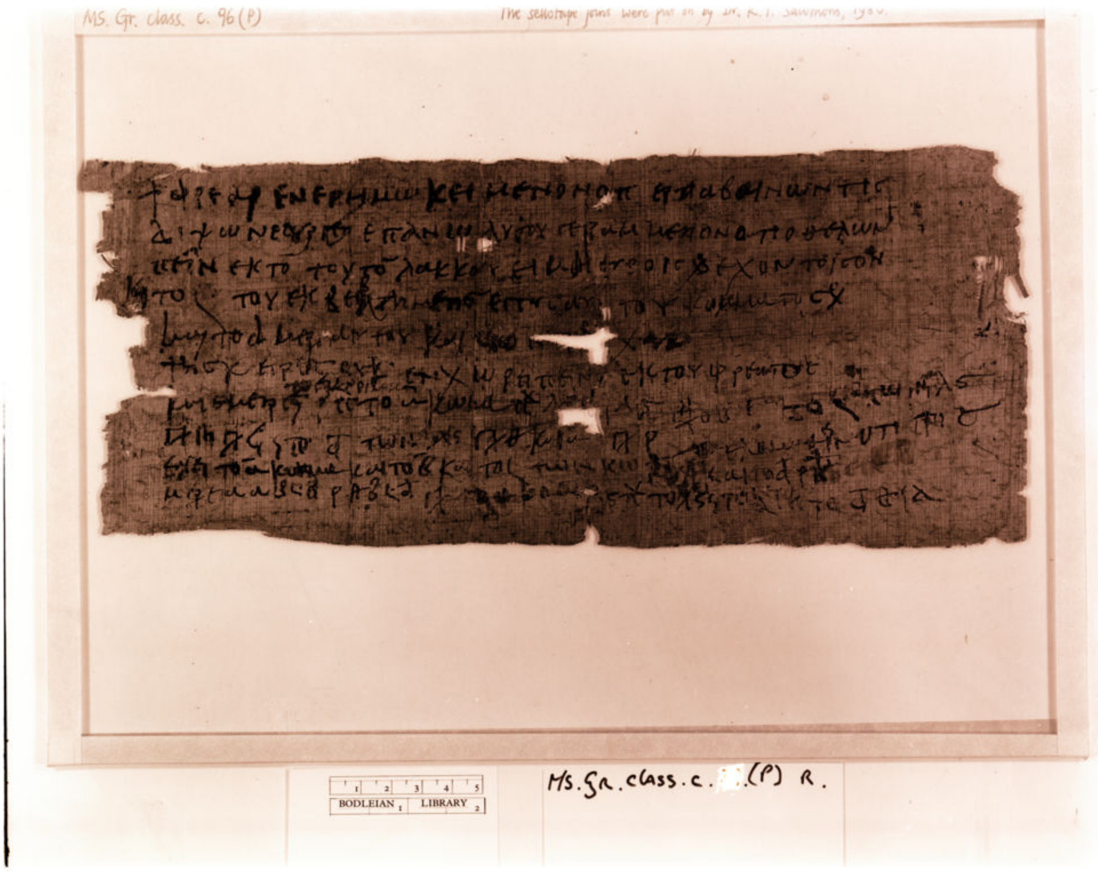

Fig. 1: The Bodleian Libraries, University of Oxford, Papyrus MS Gr. class. c 96. Creative Commons License Attribution-NonCommercial 4.0 (CC BY-NC 4.0).

† φρέαρ ἐν ἐρήμῳ κείμενον· ὃ περ⟦τι⟧αβ⟦ε̣⟧αίνων τις

διψῶν ε⟦ ̣ ⟧ὗρ⟦γ⟧εν ἐπάνω αὐτοῦ γεραμμένον ὅτι ὁ θέλων

πεῖν ἐκ τούτου τοῦ λάκκου εἰ μ⟦οι̣⟧ὴ εὕροι σχο(ινίον) ἔχον τὸ ἴσον

καὶ τὸ L τοῦ ἐκβε\β/⟦ ̣ ⟧λ⟦τ⟧ημένου ἐγγὺς αὐτοῦ κό⟦γ⟧μματο<ς > σχο(ινίου)

καὶ τὸ d μέρ(ος) αὐτοῦ καὶ ⟦ ̣ ⟧ τ̣ὼν π̣ῆ̣χυ̣ν̣

τῆς χειρὸς, οὐκ ἐνχωρεῖ πεῖν ἐκ τοῦ φρέατος·

καὶ ἐμέτρ⟦σ⟧ει \τὸ ἐξερριμμ(ένον)/ ρ π(η)χ(ῶν). τὸ $\overline{α}$ κώμα π(η)χ(ῶν) λϛ· κ̣[α]ὶ̣ λϛ (γίνεται) οβ· (καὶ) τὸ L τ̣ῶ̣ν̣ λϛ,

(γίνεται) ιη, (γίνεται) ϙ· (καὶ) τὸ đ τῶν λϛ, (γίνεται) θ, καὶ α (γίνεται) ρ. — θέλω μαθ\εῖ/⟦ι⟧ν ὅτι ⟦ ̣ ⟧ ποσ

ἔχει τὸ $\overline{α}$ κό⟦ ̣ ⟧μ̣μα καὶ τὸ $\overline{β}$ καὶ τὸ L τῶν κώ⟦ ̣ ⟧μ̣α̣ καὶ τὸ đ. ρ π(ή)χ(εις) ⟦ ̣ εἰς̣ đ⟧

μέρ(ισον) [-0-1-] α α, β, L, đ· ρ (παρὰ) β L đ· (γίνεται) τὸ α̣ κ̣ό̣μμ̣(α) λϛ π(η)χ(ῶν), τὸ \ ̣ / λϛ (καὶ) τὸ L, ιη, τὸ đ, θ, (καὶ) α.

1 *l.* παραβαίνων ὅπερ παβαίνων (pi and second alpha ex corr.) Salomons **2** εὗρεν ex εωργεν (*l.* ηὗρεν) Salomons *l.* γεγραμμένον **3** πεῖν pap., *l.* πιεῖν *l.* εὕρῃ ἴσον pap. **4** καὶ in left margin ἐκβεβλημένου ex εκβεπ̣τημενου Salomons εἰτύς *l.* εὐθύς Salomons **5** *l.* τὸν τῷ ἐ χ Salomons **6** *l.* τῇ χειρί Salomons *l.* ἐγχωρεῖ πιεῖν φρέατος ex φρεατους(?) Salomons **7** *l.* κόμμα λϛ ὁ̣(μ̣ο̣ί̣ω̣ς) λϛ Salomons **8** — pap. ελωμα θιε ν τισ ς Salomons **9** *l.* κομμάτων ⟦ εἰς̣ ∂⟧ om. Salomons **10** μέρ(ιζε) εἰ(ς) α α (ἥμισυ τέταρτον), (ἥμισυ), (τέταρτον) Salomons γί(νονται) ψ̣ο̣ί̣φοι̣ς̣ λϛ Salomons

† A well located in the desert. A thirsty person came upon it and found written on top of it that if one wants to drink from this cistern, he cannot drink from the well unless he finds a rope that, if it had one and a half times the size of a (sc. required) piece of rope extended next to it and a quarter of it and one cubit of his hand, he would measure the stretched out(?) piece to be 100 cubits. The first piece is 36 cubits; and 36, the result is 72; and half of 36 equals 18, (adding it) equals 90; and a quarter of 36, equals 9, and 1 totals 100. — I want to know how long the 1st piece and the 2nd and half of the pieces and a quarter are. Divide 100 cubits by(?) 1 (and) 1, (that is) 2, 1/2, 1/4. 100 (divided) by 2 1/2 1/4. The result is that the first piece is 36 cubits, the (second?) is 36, and a half is 18, (and) a quarter is 9; and 1.

§5 1 ὃ περ⟦τι⟧αβ⟦ε̣⟧αίνων, *l.* ὁ παραβαίνων. An alternative reading is ὅπερ τ̣ι̣αβαίνων, *l.* διαβαίνων, suggested to us in correspondence by Demokritos Kaltsas, who points out that διαβάτης in Modern Greek stands for ὁδοιπόρος, παροδίτης, ‘passer-by’. However, as Kaltsas also notes, the resulting syntax with ὅπερ would be very loose. Salomons understood ὅπερ παβαίνων for ὅπερ παραβαίνων.

§6 4 ἐκβε\β/⟦ ⟧λ⟦τ⟧ημένου: The verb ἐκβάλλω may have been used in an earlier version of the problem in the technical sense common in geometry of ‘produce’ or ‘extend’; see, for example, the definition of parallel lines in Eucl. *Elementa, Def.* 23: Παράλληλοί εἰσιν εὐθεῖαι, αἵτινες ἐν τῷ αὐτῷ ἐπιπέδῳ οὖσαι καὶ ἐκβαλλόμεναι εἰς ἄπειρον ἐφ’ ἑκάτερα τὰ μέρη ἐπὶ μηδέτερα συμπίπτουσιν ἀλλήλαις, ‘Parallel are lines that lie on the same plane and, being extended indefinitely in both directions, do not intersect in either direction’. But this sense is difficult to reconcile with the text in its current, apparently corrupted form. Alternatively, one might take the verb ἐκβάλλω to mean ‘discard’ and to refer to a piece of rope that has been discarded near the cistern (ἐγγὺς αὐτοῦ, sc. λάκκου).

§7 ἐγγύς: Salomons read εἰτύς, *l.* εἰθύς, but the second letter looks more like gamma.

§8 5–6 τ̣ῶν (*l.* τὸν) π̣ῆχ̣υ̣ν̣ τῆς χειρός: The article has the possessive meaning here, ‘a cubit of his own hand’, which is paralleled by the central character in Alcuin’s problems who adds himself to the imagined number of men in problem 2 or of sheep in 40, both discussed below.

§9 7 ἐμέτρ⟦σ̣⟧ε̣ι̣ … ρ π(η)χ(ῶν): There is no doubt that 100 cubits belongs to the statement of the problem, but the phrasing is unclear and the syntax is surely confused.

§10 τὸ ἐξερριμμ(ένον): We follow Salomons in printing this, albeit with considerable reservation. For one, the superimposed chi of π(η)χ(ῶν) interferes with the inserted word, giving what we believe is a false impression of the letter rho (the first rho in Salomons’ ἐξερριμμ(ένον)). Near the end of the inserted word there seems to be a further insertion resembling gamma (it could also be sigma) above the first mu; it is doubtful that this is just smudged ink from the lower part of kappa in the line above. From the standpoint of paleography, \τὸ ἐξῳ̣ρι\γ/ ̣μ( )/ seems to us a better reading. The letter below the gamma looks, at first glance, like mu, but the extended left-to-right ascending oblique stroke reaches farther than in any other mu. One could consider a perfect passive participle of a (hitherto unattested) compound verb ἐξοργυιόομαι (cf. ὀργυιόομαι ‘to stretch’, ‘spread out’), i.e., ἐξῳρι\γ/ωμ(ένον) for ἐξωρ\γ/ιωμ(ένον), *l.* ἐξωργυωμένον, although that too is probably a bit of a stretch.

§11 7–8 τὸ α̅ κώμα π(η)χ(ῶν) λϛ … καὶ α (γίνεται) ρ. This clause combines the answer to the problem ('the first piece is 36 cubits') with its verification, that is, a demonstration that the quantity arrived at satisfies the conditions set out in the statement of the problem.

§12 8 θέλω μαθεῖν ὅτι ποσ() … Before the first theta is an ascending paragraphos sign. The theta is large and smeared and thus difficult to recognize; it is positioned at the level of the paragraphos and thus lower than the writing before the sign, with the rest of the line ascending gradually. This is possibly why Salomons treated it as a superlinear insertion above καὶ τὸ 𐅸 ρ π(ή)χ(εις) at the end of line 9 and could not make sense of it. The expression belongs to the language of mathematical problems where it introduces the question about the unknown quantity to be determined, cf. problems 13, 17 and 49 in P.Cair.cat. 10758 (TM 64999) and P.Michael. 62, passim (TM 65020). However, one does not expect the question of the problem to be posed at this point, after its solution.

§13 The π in ποσ( ) is corrected from an assemblage consisting of π with ο written between its legs.

§14 8–9 ποσ() ἔχει τὸ α̅ κό⟦.⟧μ̣μα καὶ τὸ β̅ καὶ τὸ L τῶν κώ⟦.⟧μ̣α̣ καὶ τὸ 𐅸: The question of the size—how many (sc. cubits)—pertains not only to the length of the first rope but to all its parts (the first rope, the second, the half and the quarter).

§15 9–10 ρ π(ή)χ(εις) ⟦. εἰ̣ς̣ 𐅸⟧ | μέρ(ισον). (.) α̣ α, β, L, 𐅸 ρ (παρὰ) β L 𐅸: This is the method used to solve the problem: The hundred cubits, which is the length of the extended rope, is divided into the number of its constituent parts, which is 2 1/2 1/4. The writer evidently forgot that one cubit 'of his own hand' should first have been subtracted from the 100.

§16 The first word in line 10 is abbreviated μερ, likelier for μέρ(ισον) than for μέρ(ιζε); cf., e.g., Chester Beatty codex AC 1390 (Upper Egypt, late 3rd–first half of the 4th c.; TM 61614), where μέρισον is always spelled out. What follows the abbreviation is unclear. Salomons printed εἰ(ς), and the traces are compatible with a ligatured epsilon and iota, but there is no sign of an abbreviation. Next, parts of the rope are apparently added up, although there is no explicit instruction or record of the procedure, which is normally expressed with the terms σύνθες or συντίθω/συνθήσω. What we have looks somewhat like a brief note of an oral presentation: One can imagine that the person explicating the solution clarified that one piece corresponds to 1, and the other also to 1, and thus together they are 2, and then 1/2 and 1/4. All this is then summarized in a more formalized entry: 100 (sc. divide) by 2 1/2 1/4.

§17 ⟦.εἰ̣ς̣ 𐅸⟧: This reading seems to us better than ⟦.εἰ̣ς̣ α⟧ given the horizontal stroke which is clearly visible and must have crossed the vertical of d, as it does in nearly every other instance of the fraction. If the reading is correct, the erased operation, '(reduce) to 1/4', would have belonged to the computation, in which both the dividend and the divisor were reduced to 1/4 to make the division easier; see below. The note, however, stands in the wrong place: One expects it to come after the operation of division is stated, i.e., after ρ (παρὰ) β L 𐅸 in line 10. Perhaps the writer first skipped the calculation method and then, wanting to add it, put it in the wrong place and subsequently expunged it. Or, it could have been a marginal note—rather smudged than erased—penned after the text was completed and meant as a brief reminder that the computation should be performed by reducing to 1/4.

§18 10 (γίνεται) τὸ α̣ κ̣ό̣μ̣μ̣(α) λϛ π(η)χ(ῶν) τὸ \.̣/ λϛ (καὶ) τὸ L ιη τὸ 𐅸 θ (καὶ) α. It is not clear if, where we print τὸ \.̣/ λϛ, something was inserted or some ink was simply smudged. If there was an insertion, something like τὸ \ἴσον/ or \β̅/ would make sense, but we are not sure that the traces are consistent with either reading.

P.Bodl. 1 7 and Alcuin's *Propositiones ad acuendos iuvenes*

§19 There is little wonder that Salomons found the presentation of the problem in P.Bodl. 1 7 chaotic, as indeed it is not characteristic of word problems in Greek papyrological evidence. Constituent elements of these problems usually include a statement, often presented as a life-like situation; a question; a description of the method by which the problem is to be solved; the answer; and a verification in the form of a demonstration that the answer satisfies the conditions set out in the statement.[3] Not all of these elements are always present in any given problem, but those that are stand in this order. However, the sequence of elements in the Bodleian papyrus is different. It can be rendered schematically as follows: Statement — Answer — Verification — Question — Method — Answer. At first glance, this makes little sense, but the key to understanding what is going on is furnished by the paragraphos sign in line 8, which signals a break in the text, splitting it into two parts. The part leading up to the sign is a complete and self-contained presentation of the problem, which includes the statement, answer and verification:

§20 (*Statement*) A well located in the desert. A thirsty person came upon it and found written on top of it that if one wants to drink from this cistern, he cannot drink from the well unless he finds a rope that, if it had one and a half times the size of a (sc. required) piece of rope extended next to it and a quarter of it and one cubit of his hand, he would measure the stretched out(?) piece to be 100 cubits. (*Answer*) The first piece is 36 cubits; (*Verification*) and 36, the result is 72; and half of 36 equals 18, (adding it) equals 90; and a quarter of 36, equals 9, and 1, totals 100 (lines 1–8).

§21 This part of the text is strikingly similar to word problems conventionally labeled 'Gott Grüß Euch-Aufgaben', which owe their name to their common form as a story about a passerby meeting a company of people.[4] Widely attested in the medieval tradition, these problems ask one to find the unknown quantity from the given sum of its multiples and parts and sometimes also from an additional stated amount. The collection *Propositiones ad acuendos iuvenes* transmitted under the name of Alcuin of York and dated to c. 800 preserves two early specimens featuring the same quantities as our papyrus does.[5] Let us look at problem no. 2 of that collection (ed. Folkerts; our translation):

§22 *Propositio de viro ambulante in via* : *Quidam vir ambulans per viam vidit sibi alios homines obviantes et dixit eis: Volebam, ut fuissetis alii tantum, quanti estis, et medietas medietatis, et rursus de medietate medietas; tunc una mecum C fuissetis. Dicat, qui vult, quot fuerint, qui in primis ab illo visi sunt.*

§23 *Solutio* : *Qui imprimis ab illo visi sunt, fuerunt XXXVI. Alii tantum fiunt LXXII, medietas medietatis sunt XVIII, et huius numeri medietas sunt VIIII. Dic ergo sic: LXXII et XVIII fiunt XC. Adde VIIII, fiunt XCVIIII. Adde loquentem, et habebis C.*

§24 Problem concerning a man walking on the road : Some man walking along a road saw other men coming towards him and he said to them: 'I wish there were so many more of you as you are now; plus half of the half (sc. of the resultant sum); and again half of that (sc. last) amount. Then together with me you would be 100.' Whoever wishes can say how many men were first seen by that man.

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3 Cf. the discussion of the patterns of phrasing problems in papyri in Bagnall and Jones 2019: 21–23.

4 For a description of the type, see Tropfke 1980: 574–575 (4.2.1.1.1); van Egmond 1996: 401.

5 For an overview of 'Gott Grüß Euch-Aufgaben' in Alcuin's collection, see Folkerts 1978: 35–36.

§25 Solution : Those who were first seen by the man were 36 in number. The others, amounting to the same, make 72. Half of the half (of this) is 18, and half of this number is 9. Therefore, say this: 72 and 18 make 90. Add 9 and there will be 99. Add the speaker and you will get 100.

§26 No. 40 in the collection is the exact same except that a man sees sheep instead of other men, while a problem in a 15th-century Byzantine manuscript (Cod. Vindob. phil. gr. 65, no. 46) has a man meet dancing girls.[6] In all these problems, just as in the Bodleian papyrus, the task *in abstracto* is to find a number such that adding to it itself, its half, its quarter and 1 results in 100.

§27 Besides having the same mathematical content, the versions in the Bodleian papyrus (lines 1–8) and in Alcuin's collection as exemplified by no. 2 also have similar sets of elements and both omit the method for solving the problem.[7] The differences are largely of style: The story on the papyrus is fitting for an Egyptian environment (φρέαρ ἐν ἐρήμῳ, 'a well in the desert'), its question is not explicitly stated but implied, and the record of the answer and demonstration is very condensed.

P.Bodl. 1 7 and *ꜥḥꜥ* problems

§28 The two-and-a-half lines of text after the paragraphos in line 8 seem to defy logic: They begin by formulating the question of the problem, even though the answer to it has already been stated, and then they give the method of solving it and (again) the answer. The sequence Question—Method—Answer would be perfectly reasonable had it followed the statement directly and not the answer and verification. What seems to have happened is that the writer conceived of or was given lines 1–8 as a self-contained presentation of the problem. Since it had no solution method, he treated it as if it were only the statement; next, he added the method of solving it and the resulting answer. The sequence of elements then became what is usual for papyrological texts: Statement—Question—Method—Answer.

§29 The formulation of the question, 'I want to know how long the 1st piece and the 2nd and a half of the pieces and a quarter are', suggests that the writer viewed the problem somewhat differently from what the versions in Alcuin and in the first part of the papyrus imply. To judge from the answer given in line 7 ('the first piece is 36 cubits') and in Alcuin's version ('the men first seen were 36'), the composers of those texts saw the task as determining one unknown quantity, just as we would. The question of the problem posed after the paragraphos sign, however, pertains to the lengths of all parts of the rope, possibly indicating that for the composer of these lines the problem had several unknowns.

§30 A similar concept of computing the unknown quantity and all its stated parts is found in some of the *ꜥḥꜥ* (or *aha*) problems preserved in Egyptian papyri. In these problems, the majority of which are preserved in the Rhind mathematical papyrus, a large roll inscribed with mathematical problems and tables copied by the scribe Ahmes in ca. 1550 BC, as well as in a papyrus kept in Moscow,[8] an unknown quantity referred to as *ꜥḥꜥ* (*aha*) is to be determined on the basis of a stated transformation of it. In particular, the group of problems pRhind nos 24–27, in which a quantity and a fraction of it are added and the resulting quantity is stated, compute not only the unknown quantity but also its parts. The importance of this feature was pointed out long ago by Otto Neugebauer who, noting that 'bei den ꜥḥꜥ-Rechnungen eigentlich die Bestimmung mehrerer Unbekannten (d.h. der wirklichen einzelnen

6 Cf. Hunger and Vogel 1963: 95, where further examples are cited; Tropfke 1980: 574–575.

7 No problem in Alcuin's collection is accompanied by a solution (we note that the Latin term *solutio* in Alcuin corresponds to the answer and verification, while our 'solutions' refers to the method by which the problem was solved). The later medieval tradition attests solutions of this type of problem by application of the method of false position, as, for example, in problem 46 of Cod. Vindob. phil. gr. 65 (mentioned above), which is mathematically identical to the one in the Bodleian papyrus and to Alcuin's nos 2 and 40; see Hunger and Vogel 1963: 105.

8 For the translations and interpretation of *ꜥḥꜥ* problems in pRhind and pMoscow we follow Imhausen 2002; Imhausen 2003: 39–53.

Summanden) das Ziel der Rechnung ist'[9] emphasized the need to understand the difference between the modern interpretation and the perception of the Egyptian computer:

§31 > Daß es sich offenbar um zwei gesuchte Größen handelt, scheint mir besonders beachtenswert und für den Unterschied zwischen moderner und ägyptischer Betrachtungsweise charakteristisch. Der modernen Auffassung genügt es, die eine Unbekannte *x* zu bestimmen, welche der vorgelegten linearen Gleichung genügt; der Ägypter dagegen sucht *nach den einzelnen Summanden*, aus denen sich die gegebene rechte Seite aufbauen soll, und nennt sie demgemäß einzeln im Resultat.[10]

§32 This point has been recently elaborated on by Annette Imhausen who, while also warning against an anachronistic interpretation of *ꜥḥꜥ* problems as equivalent to linear equations with one unknown, demonstrates how algorithms used to solve them could serve as the basis for a typological distinction.[11] In other words, while all *ꜥḥꜥ* problems in pRhind might be the same for a modern observer, they were different for the scribe who copied them, depending on what question was asked and what method was used to answer it.

§33 The second part of the papyrus also contains some indication, even if not a full explication, of the method used to solve it. The procedure is described very succinctly, with no introductory clause such as 'this is how it should be done' or 'I do it this way' vel sim., nor are its stages marked. Yet, the first step is clearly the addition of all the parts of the extended rope, which is what must be meant at the beginning of line 10 (α̣ α, β, L, ð). The result of this addition serves as the divisor in the division ρ (παρὰ) β L ð, which constitutes the second step of the solution to the problem.

§34 How this division was carried out is not recorded, although traces at the end of line 9 suggest a possibly deleted note about reducing to 1/4, (ἀνάλυσον) εἰς ð, an operation that would facilitate the division with a divisor which contains a fractional part.[12] It is likely, however, that our writer did not actually compute the division, because had he done so, he might have noticed that a necessary step had been left out from his algorithm. Indeed, he forgot to subtract the one cubit from the 100 cubits of the 'extended' rope, which comprised 2 1/2 1/4 parts *and* one cubit: The dividend in the division by 2 1/2 1/4 ought to have been 99, not 100! That the given answer is correct was surely owed not to computations in which our writer disregarded the fractional part of the quotient (for, 100 ÷ 2 1/2 1/4 = 36 1/3 1/33), but to his taking the answer from the solution and verification of the problem in lines 7 to 8.

§35 The two steps recorded on the papyrus—the addition (of the parts, here 2 1/2 1/4) and the division (of the whole, here 100) by the sum of the parts—correspond to the two-step algorithm used in a number of *ꜥḥꜥ* problems in pRhind, in which an unknown quantity and a number of its parts are added and the result of the addition is given (Group 2 in Imhausen's classification, which comprises pRhind 30–34 and pMoscow 25).[13] For example, in pRhind 32, we find: 'A quantity, its 1/3 and its 1/4 (sc. added) to it so that 2 results'. Solving the problem consists of two steps, just like it does in the Bodleian papyrus: First, all parts are added together (1 + 1/3 + 1/4 = 1 1/3 1/4) and then their sum, given in the statement, is divided by the number of parts (2 ÷ 1 1/3 1/4).

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9 Neugebauer 1931: 314.

10 Neugebauer 1931: 308.

11 See Imhausen 2001; Imhausen 2002; Imhausen 2003: 35–53.

12 Reducing to $\frac{1}{n}$ of a number containing a fractional part $\frac{m}{n}$ means finding a number, one-nth of which would be an integer; in our case it would be 4. For the operation of 'reduction,' cf. problems in P.Mich. 3 145 (TM 63556) with comm. to III.v.1–4.

13 For classification of the *ꜥḥꜥ* problems, cf. Imhausen 2002 and Imhausen 2003: 39–53.

§36 In the majority of transmitted *ꜥḥꜥ* problems, there is no 'extra' quantity added to the number of parts of the unknown, which would correspond to the one cubit in the Bodleian problem. In the only case where there is, the computer is fully aware that it must be first subtracted from the given sum: This is in pMoscow 19, in which a quantity calculated one and a half times together with 4 comes to 10 (Group 3) and in the solution of which the scribe first subtracts 4 from 10. It might be that our writer—or whomever he followed—was accustomed to solving problems without that 'additional quantity', and this caused his confusion.

Between Ahmes and Alcuin

§37 As the foregoing analysis demonstrates, the Bodleian papyrus comprises two parts, one that shows striking similarities in content and structure to the 'Gott Grüß Euch-Aufgaben' of the later medieval tradition, and the other that reflects concepts and algorithms whose roots can be traced back to Pharaonic mathematical techniques. This peculiar combination was likely owed to the fact that the problem recorded in the first part of the papyrus had circulated as an independent entity, whether alone or as belonging to a collection of problems whose presentation featured the question, the answer and the verification of the answer, much as Alcuin's later collection did. The colorful scenario and 'nice' integer quantities might indicate an origin in the Hellenistic or early Roman period, when problems dressed up as stories seem to have become popular, a trend culminating in mathematical epigrams in Book 14 of the *Anthologia Palatina*.[14]

§38 By the time the writer of the Bodleian papyrus encountered it, the text of the problem must have deteriorated in the process of copying or oral transmission, losing its narrative coherence but preserving the catchiest elements—the well, the rope, the thirsty passerby, as well as the numerical values of the variables. To this he added the formulation of the task and the method of solving it. The cursory way in which he recorded the solution makes one wonder if it was the product of taking notes or of an actual—and flawed—attempt to solve the problem rather than the result of copying it from a manual. The fact that the papyrus is not a codex page or part of a roll but a separate sheet lends further support to this supposition.

§39 Our writer must have viewed the problem he had in front of him differently from how it was conceived by the original composer, or from how we would see it. For him, it had several unknown quantities, not just one, and to solve it he looked for algorithms originating in the Pharaonic tradition, which he may have been used to applying. He picked the wrong one as he probably did not quite understand them, but since he already had the right answer in the text of the problem, he must have 'adjusted' his result. As is often the case with problems in the papyrological evidence, it is the mistakes that tell more of a story than a flawless execution. The uniqueness of the Bodleian papyrus is that it preserves an elaborately presented word problem that at some point was transmitted to the Latin West where its 'scenario' was changed, while, at the same time, it documents how that very problem was handled on one particular occasion somewhere in the Egyptian countryside.

Bibliography

Folkerts, M. (1978) Die älteste mathematische Aufgabensammlung in lateinischer Sprache: Die Alcuin zugeschriebenen Propositiones ad Acuendos Iuvenes (Österreichische Akademie der Wissenschaften Mathematisch-naturwissenschaftliche Klasse, Denkschriften 116/6), Vienna.

14 For a relatively early papyrological example attesting literary efforts in formulating the statement of a problem cf. P.Vindob. G26011e, in which a statue complains to Zeus about losing parts of its weight to various agents (TM 63194; 1st c., Soknopaiou Nesos) = MPER 15 179, reedited in Lougovaya 2021.

Hunger, H., and Vogel, K. (1963) Ein byzantinisches Rechenbuch des 15. Jahrhunderts (Österreichische Akademie der Wissenschaften, Philosophisch-historische Klasse, Denkschriften 78/2), Vienna.

Imhausen, A. (2001) “Die aHa-Aufgaben der ägyptischen mathematischen Texte und ihre Lösungen,” in C.-B. Arnst, I. Hafemann and A. Lohwasser (eds), Begegnungen. Antike Kulturen im Niltal, Leipzig: 213–220.

Imhausen, A. (2002) “The Algorithmic Structure of the Egyptian Mathematical Problem Texts,” in J. Steele and A. Imhausen (eds), Under One Sky. Astronomy and Mathematics in the Ancient Near East. Proceedings of the Conference held in the British Museum, London, June 25–27, 2001 (AOAT 297), Münster: 147–166.

Imhausen, A. (2003) Ägyptische Algorithmen. Eine Untersuchung zu den mittelägyptischen mathematischen Aufgabentexten, Wiesbaden.

Lougovaya, J. (2021) “A Statue’s Weight Problem. Reedition of P.Rain.Unterricht 179,” ZPE 220: 254–258.

Neugebauer, O. (1931) “Arithmetik und Rechentechnik der Ägypter,” in O. Neugebauer, J. Stenzel and O. Toeplitz (eds), Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, Abteilung B: Studien, Bd. 1, Berlin: 301–380.

Tropfke, J. (1980) Geschichte der Elementarmathematik. Band 1: Arithmetik und Algebra. Fourth edition, revised by K. Vogel, K. Reich and H. Gericke, Berlin / New York.

van Egmond, W. (1996): “Types and traditions of mathematical problems: a challenge for historians of mathematics,” in M. Folkerts (ed), Mathematische Probleme im Mittelalter. Der lateinische und arabische Sprachbereich, Wiesbaden: 379–428.

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